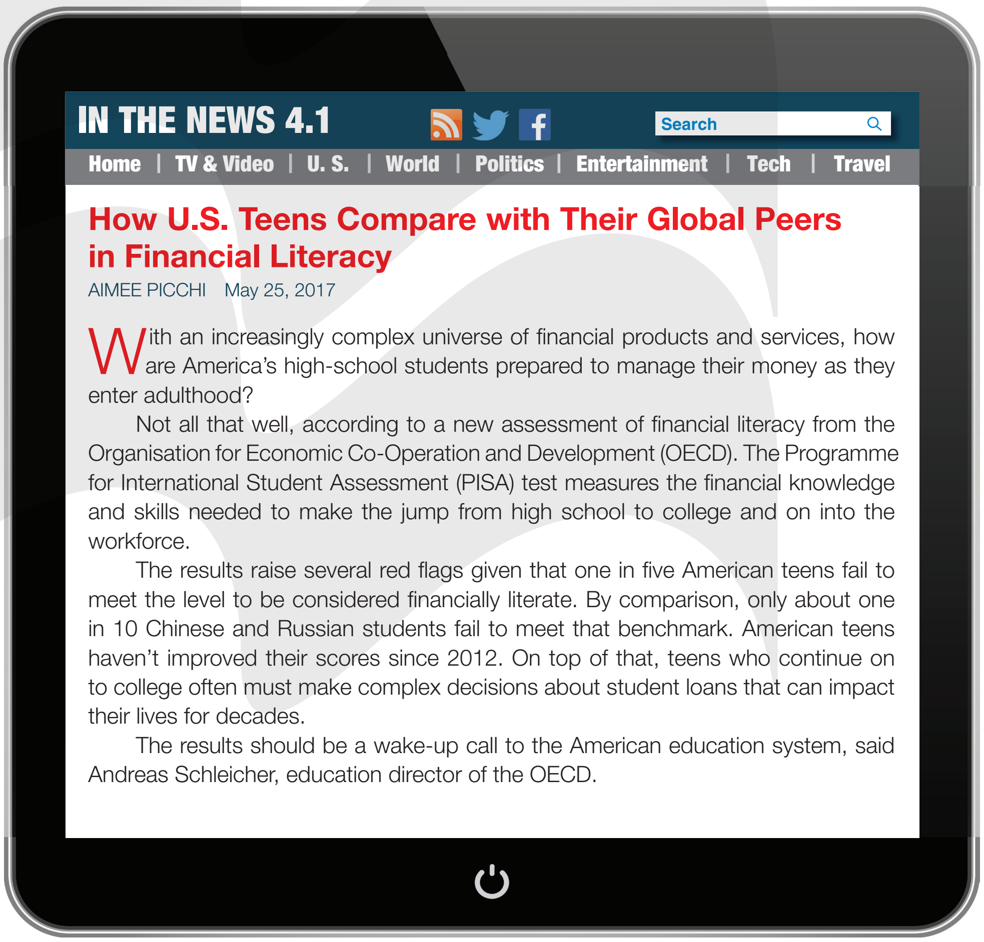
Helle, Ken/ZUMA Press/Newscom

4 Personal Finance

The following article appeared at CBSNews.com.



IN THE NEWS 4.1

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How U.S. Teens Compare with Their Global Peers in Financial Literacy

AIMEE PICCHI May 25, 2017

With an increasingly complex universe of financial products and services, how are America's high-school students prepared to manage their money as they enter adulthood?

Not all that well, according to a new assessment of financial literacy from the Organisation for Economic Co-Operation and Development (OECD). The Programme for International Student Assessment (PISA) test measures the financial knowledge and skills needed to make the jump from high school to college and on into the workforce.

The results raise several red flags given that one in five American teens fail to meet the level to be considered financially literate. By comparison, only about one in 10 Chinese and Russian students fail to meet that benchmark. American teens haven't improved their scores since 2012. On top of that, teens who continue on to college often must make complex decisions about student loans that can impact their lives for decades.

The results should be a wake-up call to the American education system, said Andreas Schleicher, education director of the OECD.

In this chapter, we explore basic financial terminology and mechanisms. In Section 4.1, we examine the basics of compound interest and savings, and in Section 4.2, we look at borrowing. In Section 4.3, we consider long-term savings plans such as retirement funds. In Section 4.4, we focus on credit cards, and in Section 4.5, we discuss financial terms heard in the daily news.

4.1 Saving money: The power of compounding

TAKE AWAY FROM THIS SECTION
Understand compound interest and the difference between APR (annual percentage rate) and APY (annual percentage yield).

The following article from CNNMoney says that many Americans are not saving enough money.

IN THE NEWS 4.2

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Americans Still Aren't Saving Enough

NEW YORK (CNNMoney)

MELANIE HICKEN @MELHICKEN February 26, 2013

Many Americans are still failing to sock away enough money to pay for retirement—or emergencies, for that matter—according to a survey of more than 1,000 people released Monday.

Only half of respondents reported good savings habits, including having a spending and/or saving plan in place, according to the Consumer Federation of America and the American Savings Education Council, which conducted the survey.

While more than half of all Baby Boomers and Gen-Xers will be able to retire with enough money to cover basic retirement needs, including health care costs, a significant number are at risk of running short, according to projections by the Employee Benefit Research Institute.

The survey also found that only 49% of non-retired respondents feel they are saving enough to achieve a "desirable standard of living" in retirement.

When it came to overall household savings, the survey found there was little improvement compared to last year.

"Clearly the great recession has had a lingering effect on many Americans," said Stephen Brobeck, executive director of the Consumer Federation of America. "Millions of families have been unable to make progress in rebuilding their finances, especially their savings."

Courtesy CNN.

Money management begins with saving, and (as the preceding article notes) Americans generally don't save enough. In this section, we see how to measure the growth of savings accounts.

Before we delve into the issue of saving money, we should say a little bit about interest rates. As you can see from the chart in **Figure 4.1**, interest rates on certificates of deposit (CDs) have fluctuated over the years, and in recent years they have been at historic lows. As of April 2021, a one-year CD was paying about 0.5% and a five-year CD 1.0%. While we want to acknowledge realistic rates, many of the examples

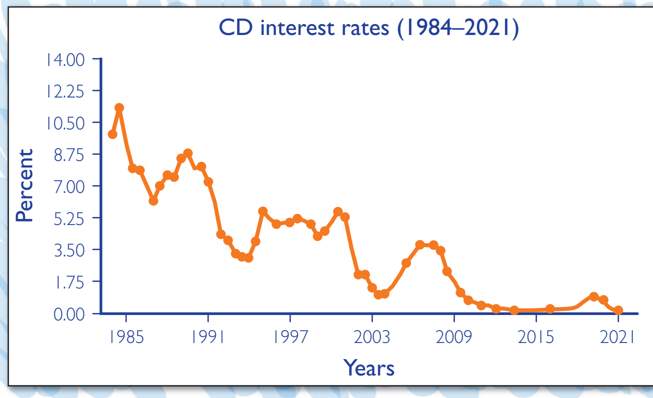


FIGURE 4.1 Certificate of deposit interest rates from 1984 to 2021.

and exercises in this chapter sometimes use rates like 6%, 10%, or 12% to help make concepts easy to understand by avoiding complex and distracting calculations.

Some readers may wish to take advantage of the following Quick Review of linear and exponential functions before proceeding.

Quick Review Linear Functions

Linear functions play an important role in the mathematics of finance.

Linear functions: A linear function has a constant growth rate, and its graph is a straight line. The growth rate of the function is also referred to as the *slope*.

We find a formula for a linear function of t using

$$\text{Linear function} = \text{Growth rate} \times t + \text{Initial value}$$

Example: If we initially have \$1000 in an account and add \$100 each year, the balance is a linear function because it is growing by a constant amount each year. After t years, the balance is

$$\text{Balance after } t \text{ years} = \$100t + \$1000$$

The graph of this function is shown in **Figure 4.2.** Note that the graph is a straight line.

For additional information, see Section 3.1 of Chapter 3.

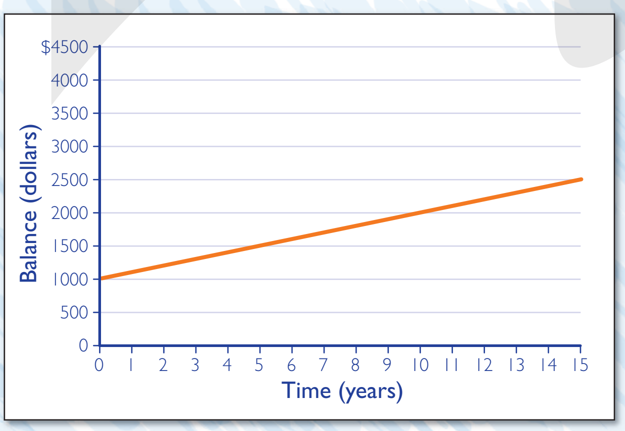


FIGURE 4.2 The graph of a linear function is a straight line.

FIGURE 4.3 The graph of an exponential function (with a base greater than 1) gets steeper as we move to the right.

Quick Review Exponential Functions

Exponential functions play an important role in the mathematics of finance.

Exponential functions: An increasing exponential function exhibits a constant percentage growth rate. If r is this percentage growth rate per period expressed as a decimal, the base of the exponential function is $1 + r$. We find a formula for an exponential function of the number of periods t using

$$\text{Exponential function} = \text{Initial value} \times (1 + r)^t$$

Example: If we initially have \$1000 in an account that grows by 10% per year, the balance is exponential because it is growing at a constant percentage rate. Now 10% as a decimal is $r = 0.10$, so $1 + r = 1.10$. After t years, the balance is

$$\text{Balance after } t \text{ years} = \$1000 \times 1.10^t$$

The graph of this function is shown in **Figure 4.3**. The increasing growth rate is typical of increasing exponential functions.

For additional information, see Section 3.2 of Chapter 3.

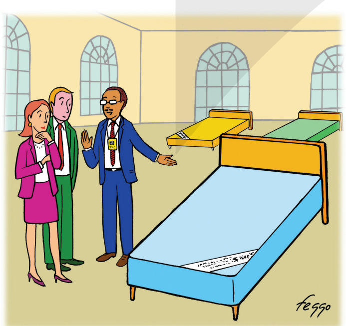
Investments such as savings accounts earn interest over time. Interest can be earned in different ways, and that affects the growth in value of an investment.

Simple interest

The easiest type of interest to understand and calculate is *simple interest*.

KEY CONCEPT

The initial balance of an account is the principal. Simple interest is calculated by applying the interest rate to the principal only, not to interest earned.



cartoonstock.com

"This mattress fits up to \$120,000 dollars."

Suppose, for example, that we invest \$1000 in an account that earns simple interest at a rate of 10% per year. Then we earn 10% of \$1000 or \$100 in interest each year. If we hold the money for six years, we get \$100 in interest each year for six years. That comes to \$600 interest. If we hold it for only six months, we get a half-year's interest or \$50.

FORMULA 4.1 Simple Interest Formula

$$\text{Simple interest earned} = \text{Principal} \times \text{Yearly interest rate (as a decimal)} \times \text{Time in years}$$

EXAMPLE 4.1 Calculating simple interest: An account

We invest \$2000 in an account that pays simple interest of 4% each year. Find the interest earned after five years.

SOLUTION

The interest rate of 4% written as a decimal is 0.04. The principal is \$2000, and the time is 5 years. We find the interest by using these values in the simple interest formula (Formula 4.1):

$$\begin{aligned}\text{Simple interest earned} &= \text{Principal} \times \text{Yearly interest rate} \times \text{Time in years}\\ &= \$2000 \times 0.04/\text{year} \times 5 \text{ years} = \$400\end{aligned}$$

TRY IT YOURSELF 4.1

We invest \$3000 in an account that pays simple interest of 3% each year. Find the interest earned after six years.

The answer is provided at the end of this section.

Compound interest

Situations involving simple interest are fairly rare and usually occur when money is loaned or borrowed for a short period of time. More often, interest payments are made in periodic installments during the life of the investment. The interest payments are credited to the account periodically, and future interest is earned not only on the original principal but also on the interest earned to date. This type of interest calculation is referred to as *compounding*.

KEY CONCEPT

Compound interest means that accrued interest is periodically added to the account balance. This interest is paid on the principal and on the interest that the account has already earned. In short, compound interest includes *interest on the interest.*

An illustration of growth due to compound interest.

To see how compound interest works, let's return to the \$1000 investment earning 10% per year we looked at earlier, but this time let's assume that the interest is compounded annually (at the end of each year). At the end of the first year, we earn 10% of \$1000 or \$100—the same as with simple interest. When interest is compounded, we add this amount to the balance, giving a new balance of \$1100. At the end of the second year, we earn 10% interest on the \$1100 balance:

$$\text{Second year's interest} = 0.10 \times \$1100 = \$110$$

This amount is added to the balance, so after two years the balance is

$$\text{Balance after 2 years} = \$1100 + \$110 = \$1210$$

For comparison, we can use the simple interest formula to find out the simple interest earned after two years:

$$\begin{aligned}\text{Simple interest after two years} &= \text{Principal} \times \text{Yearly interest rate} \times \text{Time in years}\\ &= \$1000 \times 0.10/\text{year} \times 2 \text{ years} = \$200\end{aligned}$$

The interest earned is \$200, so the balance of the account is \$1200.

After two years, the balance of the account earning simple interest is only $1200, but the balance of the account earning compound interest is $1210. Compound interest is always more than simple interest, and this observation suggests a rule of thumb for estimating the interest earned.

RULE OF THUMB 4.1 Estimating Interest

Interest earned on an account with compounding is always at least as much as that earned from simple interest. If the money is invested for a short time, simple interest can be used as a rough estimate.

The following table compares simple interest and annual compounding over various periods. It uses $1000 for the principal and 10% for the annual rate. This table shows why compounding is so important for long-term savings.

	Simple interest			Yearly compounding		
End of year	Interest	Balance	Growth	Interest	Balance	Growth
1	10% of $1000 = $100	$1100	$100	10% of $1000 = $100	$1100	$100
2	10% of $1000 = $100	$1200	$100	10% of $1100 = $110	$1210	$110
3	10% of $1000 = $100	$1300	$100	10% of $1210 = $121	$1331	$121
10	$100	$2000		$235.79	$2593.74	
50	$100	$6000		$10,671.90	$117,390.85	

To better understand the comparison between simple and compound interest, observe that for simple interest the balance is growing by the same *amount*, $100, each year. This means that the balance for simple interest is showing linear growth. For compound interest, the balance is growing by the same *percent*, 10%, each year. This means that the balance for compound interest is growing exponentially. The graphs of the account balances are shown in **Figure 4.4**. The widening gap between the two graphs shows the power of compounding.

FIGURE 4.4 Balance for simple interest is linear, and balance for compound interest is exponential.

EXAMPLE 4.2 Calculating compound interest: Annual compounding

You invest \$500 in an account that pays 6% compounded annually. What is the account balance after two years?

SOLUTION

Now 6% expressed as a decimal is 0.06. The first year's interest is 6% of \$500:

$$\text{First year's interest} = 0.06 \times \$500 = \$30.00$$

This interest is added to the principal to give an account balance at the end of the first year of \$530.00. We use this figure to calculate the second year's interest:

$$\text{Second year's interest} = 0.06 \times \$530.00 = \$31.80$$

We add this to the balance to find the balance at the end of two years:

$$\text{Balance after 2 years} = \$530.00 + \$31.80 = \$561.80$$

TRY IT YOURSELF 4.2

Find the balance of this account after four years.

The answer is provided at the end of this section.

Other compounding periods and the APR

Interest may be compounded more frequently than once a year. For example, compounding may occur semi-annually, in which case the *compounding period* is half a year. Compounding may also be done quarterly, monthly, or even daily (**Figure 4.5**). To calculate the interest earned, we need to know the *period interest rate*.

KEY CONCEPT

The period interest rate is the interest rate for a given compounding period (for example, a month). Financial institutions report the annual percentage rate or APR. To calculate this, they multiply the period interest rate by the number of periods in a year.

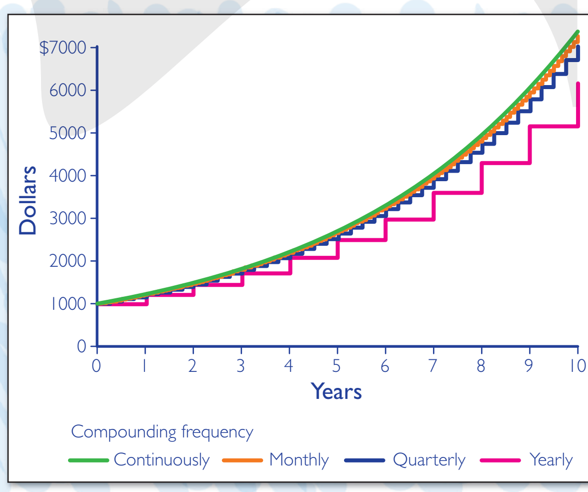


FIGURE 4.5 Effect of compounding at various frequencies at 20% APR and an initial investment of \$1000.

The following formula shows how the APR is used in calculations to determine interest rates.

FORMULA 4.2 Period Interest Rate Formula

$$\text{Period interest rate} = \frac{\text{APR}}{\text{Number of periods in a year}}$$

Suppose, for example, that we invest \$500 in a savings account that has an APR of 6% and compounds interest monthly. Then there are 12 compounding periods each year. We find the monthly interest rate using

$$\text{Monthly interest rate} = \frac{\text{APR}}{12} = \frac{6\%}{12} = 0.5\%$$

"We can give you a 12% rate if you never withdraw it."

Each month we add 0.5% interest to the current balance. The following table shows how the account balance grows over the first few months:

End of month	Interest earned	New balance	Percent increase
1	0.5% of \$500.00 = \$2.50	\$502.50	0.5%
2	0.5% of \$502.50 = \$2.51	\$505.01	0.5%
3	0.5% of \$505.01 = \$2.53	\$507.54	0.5%
4	0.5% of \$507.54 = \$2.54	\$510.08	0.5%

Compound interest formula

So far we have calculated the interest step-by-step to see how the balance grows due to compounding. Now we simplify the process by giving a formula for the balance.

If r is the period interest rate expressed as a decimal, we find the balance after t periods using the following:

FORMULA 4.3 Compound Interest Formula

$$\textbf{Balance after } t \textbf{ periods} = \textbf{Principal} \times (1 + r)^t$$

Alternatively, we can write the formula as

$$\text{Balance after } y \text{ years} = \text{Principal} \times \left(1 + \frac{\text{APR}}{n}\right)^{(n\times y)}$$

where interest is compounded n times per year and y is the number of years. This form is equivalent because $r = \dfrac{\text{APR}}{n}$ and the number of periods is $t = n \times y$.

Here is an explanation for the formula: Over each compounding period, the balance grows by the same percentage, so the balance is an exponential function of t. That percentage growth is r as a decimal, and the initial value is the principal. Using these values in the standard exponential formula

$$\text{Exponential function} = \text{Initial value} \times (1 + r)^t$$

gives the compound interest formula.

Let's find the formula for the balance if \$500 is invested in a savings account that pays an APR of 6% compounded monthly. The APR as a decimal is 0.06, so in decimal form the monthly rate is $r = 0.06/12 = 0.005$. Thus, $1 + r = 1.005$. By Formula 4.3,

$$\begin{aligned} \text{Balance after } t \text{ months} &= \text{Principal} \times (1 + r)^t \\ &= \$500 \times 1.005^t \end{aligned}$$

We can use this formula to find the balance of the account after five years. Five years is 60 months, so we use $t = 60$ in the formula:

$$\text{Balance after 60 months} = \$500 \times 1.005^{60} = \$674.43$$

The APR by itself does not determine how much interest an account earns. The number of compounding periods also plays a role, as the next example illustrates.

EXAMPLE 4.3 Calculating values with varying compounding periods: Value of a CD

Suppose we invest \$10,000 in a five-year certificate of deposit (CD) that pays an APR of 6%.

a. What is the value of the mature CD if interest is compounded annually? (*Maturity* refers to the end of the life of a CD. In this case, maturity occurs at five years.)

b. What is the value of the mature CD if interest is compounded quarterly?

c. What is the value of the mature CD if interest is compounded monthly?

d. What is the value of the mature CD if interest is compounded daily?

e. Compare your answers from parts a–d.

SOLUTION

a. The annual compounding rate is the same as the APR. Now 6% as a decimal is $r = 0.06$. We use $1 + r = 1.06$ and $t = 5$ years in the compound interest formula (Formula 4.3):

$$\begin{aligned} \text{Balance after 5 years} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times 1.06^5 \\ &= \$13{,}382.26 \end{aligned}$$

b. Again, we use the compound interest formula. To find the quarterly rate, we divide the APR by 4. The APR as a decimal is 0.06, so as a decimal, the quarterly rate is

$$r = \text{Quarterly rate} = \frac{\text{APR}}{4} = \frac{0.06}{4} = 0.015$$

Thus, $1 + r = 1.015$. Also five years is 20 quarters, so we use $t = 20$ in the compound interest formula:

$$\begin{aligned} \text{Balance after 20 quarters} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times 1.015^{20} \\ &= \$13{,}468.55 \end{aligned}$$

c. This time we want the monthly rate, so we divide the APR by 12:

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.06}{12} = 0.005$$

Also, five years is 60 months, so

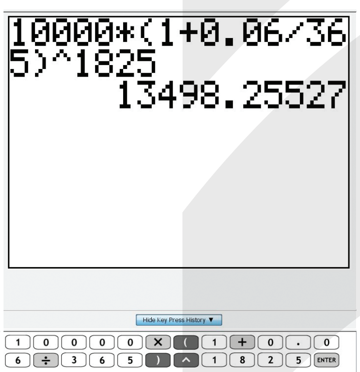
$$\begin{aligned} \text{Balance after 60 months} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times 1.005^{60} \\ &= \$13{,}488.50 \end{aligned}$$

d. We assume that there are 365 days in each year, so as a decimal, the daily rate is

$$r = \text{Daily rate} = \frac{\text{APR}}{365} = \frac{0.06}{365}$$

This is $r = 0.00016$ to five decimal places, but for better accuracy, we won't round this number. (See Calculation Tip 1.) Five years is $5 \times 365 = 1825$ days, so

$$\begin{aligned} \text{Balance after 1825 days} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times \left(1 + \frac{0.06}{365}\right)^{1825} \\ &= \$13{,}498.26 \end{aligned}$$



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e. We summarize these results in the following table:

Compounding period	Balance at maturity
Yearly	\$13,382.26
Quarterly	\$13,468.55
Monthly	\$13,488.50
Daily	\$13,498.26

This table shows that increasing the number of compounding periods increases the interest earned even though the APR and the number of years stay the same.

CALCULATION TIP 4.1 Rounding

Some financial calculations are very sensitive to rounding. To obtain accurate answers, when you use a calculator, it is better to keep all the decimal places rather than to enter parts of the formula that you have rounded. You can do this by either entering the complete formula or using the memory key on your calculator to store numbers with lots of decimal places.

For instance, in part d of the preceding example, we found the balance after 1825 days to be \$13,498.26. But if we round the daily rate to 0.00016, we get $\$10{,}000 \times 1.00016^{1825} = \$13{,}390.72$. As this shows, rounding significantly affects the accuracy of the answer.

More information on rounding is given in Appendix 3.

SUMMARY 4.1
Compound Interest

1. With compounding, accrued interest is periodically added to the balance. As a result interest is earned each period on both the principal and whatever interest has already accrued.
2. Financial institutions advertise the annual percentage rate (APR).
3. If interest is compounded n times per year, to find the period interest rate, we divide the APR by n:

$$\text{Period rate} = \frac{\text{APR}}{n}$$

4. We can calculate the account balance after t periods using the compound interest formula (Formula 4.3):

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1 + r)^t$$

Here, r is the period interest rate expressed as a decimal.

Alternatively, we can write the formula as

$$\text{Balance after } y \text{ years} = \text{Principal} \times \left(1 + \frac{\text{APR}}{n}\right)^{(n \times y)}$$

where interest is compounded n times per year and y is the number of years.

Many financial formulas, including this one, are sensitive to round-off error, so it is best to do all the calculations and then round.

APR versus APY

Suppose we see one savings account offering an APR of 4.32% compounded quarterly and another one offering 4.27% compounded monthly. What we really want to know is which one pays us more money at the end of the year. This is what the *annual percentage yield* or APY tells us.

KEY CONCEPT

The annual percentage yield or APY is the actual percentage return earned in a year. Unlike the APR, the APY tells us the actual percentage growth per year, including returns on investment due to compounding.

A federal law passed in 1991 requires banks to disclose the APY.

To understand what the APY means, let's look at a simple example. Suppose we invest \$100 in an account that pays 10% APR compounded semi-annually. We want to see how much interest is earned in a year. The period interest rate is

$$\frac{\text{APR}}{2} = \frac{10\%}{2} = 5\%$$

so we take $r = 0.05$. The number of periods in a year is $t = 2$. By the compound interest formula (Formula 4.3), the balance at the end of one year is

$$\text{Principal} \times (1 + r)^t = \$100 \times 1.05^2 = \$110.25$$

We have earned a total of \$10.25 in interest. As a percentage of \$100, that is 10.25%. This number is the APY. It is the actual percent interest earned over the period of one year. The APY is always at least as large as the APR.

"A FIVE YEAR CD? WHO'S GOT TIME TO LISTEN TO THAT?!"

Here is a formula for the APY.

FORMULA 4.4 APY Formula

$$\text{APY} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1$$

Here both APY and APR are in decimal form, and n is the number of compounding periods per year.

A derivation of this formula is shown in Algebraic Spotlight 4.1. Let's apply this formula in the preceding example with 10% APR compounded semi-annually. Compounding is semi-annual, so the number of periods is $n = 2$, and the APR as a decimal is 0.10:

$$\begin{aligned} \text{APY} &= \left(1 + \frac{\text{APR}}{n}\right)^n - 1 \\ &= \left(1 + \frac{0.10}{2}\right)^2 - 1 \\ &= 0.1025 \end{aligned}$$

As a percent, this is 10.25%—the same as we found above.

ALGEBRAIC SPOTLIGHT 4.1 Calculating APY

Suppose we invest money in an account that is compounded n times per year. Then the period interest rate is

$$\text{Period rate} = \frac{\text{APR}}{n}$$

The first step is to find the balance after one year. Using the compound interest formula (Formula 4.3), we find the balance to be

$$\text{Balance after 1 year} = \text{Principal} \times \left(1 + \frac{\text{APR}}{n}\right)^n$$

How much money did we earn? We earned

$$\text{Balance minus Principal} = \text{Principal} \times \left(1 + \frac{\text{APR}}{n}\right)^n - \text{Principal}$$

If we divide these earnings by the amount we started with, namely, the principal, we get the percentage increase:

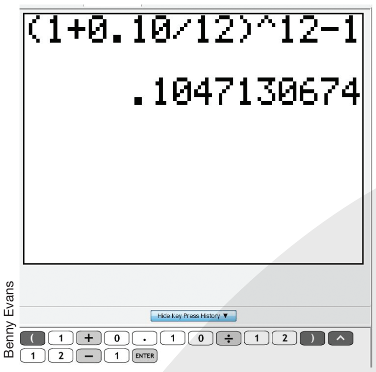
$$\text{APY as a decimal} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1$$

EXAMPLE 4.4 Calculating APY: An account with monthly compounding

We have an account that pays an APR of 10%. If interest is compounded monthly, find the APY. Round your answer as a percentage to two decimal places.

SOLUTION

We use the APY formula (Formula 4.4). As a decimal, 10% is 0.10, and there are $n = 12$ compounding periods in a year. Therefore,



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$$\begin{aligned} \text{APY} &= \left(1 + \frac{\text{APR}}{n}\right)^n - 1 \\ &= \left(1 + \frac{0.10}{12}\right)^{12} - 1 \end{aligned}$$

To four decimal places, this is 0.1047. Thus, the APY is about 10.47%.

TRY IT YOURSELF 4.4

We have an account that pays an APR of 10%. If interest is compounded daily, find the APY. Round your answer as a percentage to two decimal places.

The answer is provided at the end of this section.

Using the APY

We can use the APY as an alternative to the APR for calculating compound interest. **Table 4.1** gives both the APR and the APY for CDs from First Command

TABLE 4.1 Rates from First Command Bank (2008–2009)

CD rates with quarterly compounding[1]		
30 Day	APR	APY
$1000 – $99,999.99	3.21%	3.25%
$100,000+	3.26%	3.30%
90 Day	APR	APY
$1000 – $9999.99	3.22%	3.25%
$10,000 – $99,999.99	3.26%	3.30%
$100,000+	3.35%	3.40%
1 Year	APR	APY
$1000 – $9999.99	3.31%	3.35%
$10,000 – $99,999.99	3.35%	3.40%
$100,000+	3.55%	3.60%
18 Month	APR	APY
$1000 – $9999.99	3.56%	3.60%
$10,000 – $99,999.99	3.59%	3.65%
$100,000+	3.74%	3.80%
2 Year	APR	APY
$1000 – $9999.99	3.80%	3.85%
$10,000 – $99,999.99	3.84%	3.90%
$100,000+	3.98%	4.05%

[1] *According to federal regulations, the tolerance of 1/8 of 1 percentage point above or below the annual percentage rate applies to any required disclosure of the annual percentage rate. See* `https://www.fdic.gov/regulations/laws/rules/6500-2280.html`.

Bank during late 2008 to early 2009. We use these old rates because they demonstrate the point we are trying to make better than the current lower rates do.

The APY can be used directly to calculate a year's worth of interest. For example, suppose we purchase a one-year \$100,000 CD from First Command Bank. Table 4.1 gives the APY for this CD as 3.60%. So after one year the interest we will earn is

$$\text{One year's interest} = 3.60\% \text{ of Principal} = 0.036 \times \$100{,}000 = \$3600$$

EXAMPLE 4.5 Using APY to find the value of a CD

a. Suppose we purchase a one-year CD from First Command Bank for \$25,000. According to Table 4.1, what is the value of the CD at the end of the year?

b. In March 2014, GE Capital Retail Bank advertised a one-year \$25,000 CD at 1.04% compounded daily. They said this was an APY of 1.05%. If we purchased this one-year CD for \$25,000, what would its value be at the end of the year?

SOLUTION

a. According to Table 4.1, the APY for this CD is 3.40%. This means that the CD earns 3.40% interest over the period of one year. Therefore,

$$\text{One year's interest} = 3.4\% \text{ of Principal} = 0.034 \times \$25{,}000 = \$850$$

We add this to the principal to find the balance:

$$\text{Value after 1 year} = \$25{,}000 + \$850 = \$25{,}850$$

b. The APY for this CD is 1.05%, meaning the CD earns 1.05% interest over the period of one year. Therefore,

$$\text{One year's interest} = 1.05\% \text{ of Principal} = 0.0105 \times \$25{,}000 = \$262.50$$

We add this to the principal to find the balance:

$$\text{Value after 1 year} = \$25{,}000 + \$262.50 = \$25{,}262.50$$

TRY IT YOURSELF 4.5

Suppose we purchase a one-year CD from First Command Bank for \$125,000. According to Table 4.1, what is the value of the CD at the end of the year?

The answer is provided at the end of this section.

We can also use the APY rather than the APR to calculate the balance over several years if we wish. The APY tells us the actual percentage growth per year, including interest we earned during the year due to periodic compounding.[2] Thus, we can think of compounding annually (regardless of the actual compounding period) using the APY as the annual interest rate. Once again, the balance is an exponential function. In this formula, the APY is in decimal form.

FORMULA 4.5 APY Balance Formula

$$\text{Balance after } y \text{ years} = \text{Principal} \times (1 + \text{APY})^y$$

[2] *The idea of an APY normally applies only to compound interest. It is not used for simple interest.*

EXAMPLE 4.6 Using APY balance formula: CD balance at maturity

In April 2021, a bank offered a five-year CD at 1.09% APY. If you buy a \$100,000 CD, calculate the balance at maturity.

SOLUTION

The APY in decimal form is 0.0109 and the CD matures after five years, so we use $y = 5$ in the APY balance formula (Formula 4.5):

$$\begin{aligned}\text{Balance after 10 years} &= \text{Principal} \times (1 + \text{APY})^y \\ &= \$100{,}000 \times 1.0109^5 \\ &= \$105{,}570.11\end{aligned}$$

TRY IT YOURSELF 4.6

Suppose we earn 1.07% APY on a six-year \$50,000 CD. Calculate the balance at maturity.

The answer is provided at the end of this section.

SUMMARY 4.2
APY

1. The APY gives the true (effective) annual interest rate. It takes into account money earned due to compounding.
2. If n is the number of compounding periods per year,

$$\text{APY} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1$$

Here, the APR and APY are both in decimal form.

3. The APY is always at least as large as the APR. When interest is compounded annually, the APR and APY are equal. When compounding is more frequent, the APY is larger than the APR. The more frequent the compounding, the greater the difference.
4. The APY can be used to calculate the account balance after y years:

$$\text{Balance after } t \text{ years} = \text{Principal} \times (1 + \text{APY})^y$$

Here, the APY is in decimal form.

Future and present value

Often we invest with a goal in mind, for example, to make a down payment on the purchase of a car. The amount we invest is called the *present value*. The amount the account is worth after a certain period of time is called the *future value* of the original investment. Sometimes we know one of these two and would like to calculate the other.

KEY CONCEPT

The present value of an investment is the amount we initially invest. The future value is the value of that investment at some specified time in the future.

If the account grows only by compounding each period at a constant interest rate after we make an initial investment, then the present value is the principal, and the future value is the balance given by the compound interest formula (Formula 4.3):

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1 + r)^t$$

so

$$\text{Future value} = \text{Present value} \times (1 + r)^t$$

We can rearrange this formula to obtain

$$\text{Present value} = \frac{\text{Future value}}{(1 + r)^t}$$

In these formulas, t is the total number of compounding periods, and r is the period interest rate expressed as a decimal.

Which of these two formulas we should use depends on what question we are trying to answer. By the way, if we make regular deposits into the account, then the formulas are much more complicated. We will examine that situation in Section 4.3.

EXAMPLE 4.7 Calculating present and future value: Investing for a car

You would like to have \$10,000 to buy a car three years from now. How much would you need to invest now in a savings account that pays an APR of 9% compounded monthly?

SOLUTION

In this problem, we know the future value (\$10,000) and would like to know the present value. The monthly rate is $r = 0.09/12 = 0.0075$ as a decimal, and the number of compounding periods is $t = 36$ months. Thus,

$$\begin{aligned}\text{Present value} &= \frac{\text{Future value}}{(1 + r)^t}\\ &= \frac{\$10{,}000}{1.0075^{36}}\\ &= \$7641.49\end{aligned}$$

Therefore, you should invest \$7641.49 now.

TRY IT YOURSELF 4.7

Find the future value of an account after four years if the present value is \$900, the APR is 8%, and interest is compounded quarterly.

The answer is provided at the end of this section.

Doubling time for investments

Exponential functions eventually get very large. This means that even a modest investment today in an account that pays compound interest will grow to be very large in the future. In fact, your money will eventually double and then double again.

In Section 3.3 of Chapter 3, we used logarithms to find the doubling time. Now we introduce a quick way of estimating the doubling time called the *Rule of 72*.

KEY CONCEPT

The Rule of 72 says that the doubling time in years is about 72/APR. Here, the APR is expressed as a percentage, not as a decimal. The estimate is fairly accurate if the APR is 15% or less.

SUMMARY 4.3
Doubling Time Revisited

- The exact doubling time is given by the formula

$$\text{Number of periods to double} = \frac{\log 2}{\log(\text{Base})} = \frac{\log 2}{\log(1+r)}$$

Here, r is the period interest rate as a decimal.

- We can approximate the doubling time using the *Rule of 72*:

$$\text{Estimate for doubling time} = \frac{72}{\text{APR}}$$

Here, the APR is expressed as a percentage, not as a decimal, and time is measured in years. The estimate is fairly accurate if the APR is 15% or less, but we emphasize that this is only an approximation.

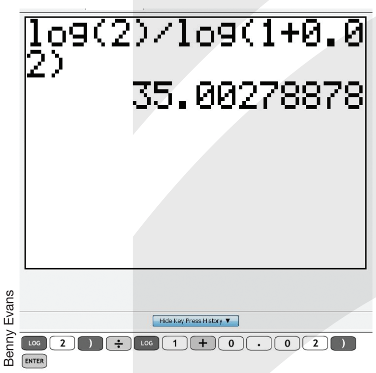
EXAMPLE 4.8 Computing doubling time: An account with quarterly compounding

Suppose an account earns an APR of 8% compounded quarterly. First estimate the doubling time using the Rule of 72. Then calculate the exact doubling time and compare the result with your estimate.

SOLUTION

The Rule of 72 gives the estimate

$$\text{Estimate for doubling time} = \frac{72}{\text{APR}} = \frac{72}{8} = 9 \text{ years}$$



Benny Evans

To find the exact doubling time, we need the period interest rate r. The period is a quarter, so $r = 0.08/4 = 0.02$. Putting this result into the doubling time formula, we find

$$\begin{aligned}\text{Number of periods to double} &= \frac{\log 2}{\log(1+r)} \\ &= \frac{\log 2}{\log(1+0.02)}\end{aligned}$$

The result is about 35.0. Therefore, the actual doubling time is 35.0 quarters, or eight years and nine months. Our estimate of nine years was three months too high.

TRY IT YOURSELF 4.8

Suppose an account has an APR of 12% compounded monthly. First estimate the doubling time using the Rule of 72, and then calculate the exact doubling time in years and months.

The answer is provided at the end of this section.

WHAT DO YOU THINK?

APR and APY: In this section we discussed the APR and the APY. A fellow student says that the APY gives a better indication of the return you can expect on an investment. Do you agree? Explain.

Questions of this sort may invite many correct responses. We present one possibility: The APY is indeed a better indication of the return you can expect on an investment than is the APR, just as it is a better indication of the interest you will pay on a consumer loan. The APR does not take into account the number of compounding periods, so this figure alone cannot tell you the interest that will be earned. The APY uses both the APR and the number of compounding periods to give the actual percentage earnings for an investment. The APY is always at least as large as the APR.

Further questions of this type may be found in the *What Do You Think?* section of exercises.

Try It Yourself answers

Try It Yourself 4.1: Calculating simple interest: An account $540.

Try It Yourself 4.2: Calculating compound interest: Annual compounding $631.24.

Try It Yourself 4.4: Calculating APY: An account with daily compounding 10.52%.

Try It Yourself 4.5: Using APY to find the value of a CD $129,500.

Try It Yourself 4.6: Using APY balance formula: CD balance at maturity $53,297.10.

Try It Yourself 4.7: Calculating present and future value: An account with quarterly compounding $1235.51.

Try It Yourself 4.8: Computing doubling time: An account with monthly compounding The Rule of 72 gives an estimate of six years. The exact formula gives 69.7 months, or about five years and 10 months.

Exercise Set 4.1

Test Your Understanding

1. True or false: Simple interest means your money earns the same amount no matter how long it stays in an account.

2. True or false: Simple interest never earns more than compound interest.

3. To say that interest is compounded means ____________.

4. To find the APR, we multiply the period interest rate by ____________.

5. True or false: Under some conditions, the APR can be more than the APY.

6. True or false: To say the interest earned by a savings account is compounded semi-annually means that equal amounts of money are deposited twice during the year.

7. The Rule of 72 tells you approximately: **a.** how much money you will have after 72 months **b.** how long it takes money earning compound interest to double **c.** what interest rate you need to double your money **d.** how much money you will have in a year if you invest 72 dollars.

8. The principal for a savings account is: **a.** the amount that you will eventually have in your account **b.** the interest rate **c.** the person who sets up the account **d.** the initial amount that is deposited.

Problems

In exercises for which you are asked to calculate the APR or APY as the final answer, round your answer as a percentage to two decimal places.

9. If the interest earned by a savings account paying an APR of 8% is compounded quarterly, what percent of the current balance is deposited every three months during the year? **a.** 2% **b.** 4% **c.** 6% **d.** 8%

10. If an investment pays an APR of 12% compounded monthly, what percentage of the current balance is added to the investment each month?

11. Simple interest. We invest $4000 in an account that pays simple interest of 3% each year. How much interest is earned after 10 years?

12. Compound interest. You invest $1000 in an account that pays 5% compounded annually. What is the balance after two years?

13. Compounding using different periods. You invest $2000 in an account that pays an APR of 6%.

a. What is the value of the investment after three years if interest is compounded yearly? Round your answer to the nearest cent.

b. What is the value of the investment after three years if interest is compounded monthly? Round your answer to the nearest cent.

14. Calculating APY. Find the APY for an account that pays an APR of 12% if interest is compounded monthly.

15. Simple interest. Assume that a three-month CD purchased for $2000 pays simple interest at an annual rate of 10%. How much total interest does it earn? What is the balance at maturity?

16. More simple interest. Assume that a 30-month CD purchased for $3000 pays simple interest at an annual rate of 5.5%. How much total interest does it earn? What is the balance at maturity?

17. Make a table. Suppose you put $3000 in a savings account at an APR of 8% compounded quarterly. Fill in the table below. (Calculate the interest and compound it each quarter rather than using the compound interest formula.)

Quarter	Interest earned	Balance
		$3000.00
1	$	$
2	$	$
3	$	$
4	$	$

18. Make another table. Suppose you put $4000 in a savings account at an APR of 6% compounded monthly. Fill in the table below. (Calculate the interest and compound it each month rather than using the compound interest formula.)

Month	Interest earned	Balance
		$4000.00
1	$	$
2	$	$
3	$	$

19. Compound interest calculated. Assume that we invest $2000 for one year in a savings account that pays an APR of 10% compounded quarterly.

a. Make a table to show how much is in the account at the end of each quarter. (Calculate the interest and compound it each quarter rather than using the compound interest formula.)

b. Use your answer to part a to determine how much total interest the account has earned after one year.

c. Compare the earnings to what simple interest or semi-annual compounding would yield.

20. Using the compound interest formula. *This is a continuation of Exercise 19.* In Exercise 19, we invested $2000 for one year in a savings account with an APR of 10% compounded quarterly. Apply the compound interest formula (Formula 4.3) to see whether it gives the answer for the final balance obtained in Exercise 19.

21. The difference between simple and compound interest. Suppose you invest $1000 in a savings account that pays an APR of 0.6%. If the account pays simple interest, what is the balance in the account after 20 years? If interest is compounded monthly, what is the balance in the account after 20 years?

22. Calculating interest. Assume that an investment of $7000 earns an APR of 6% compounded monthly for 18 months.

a. How much money is in your account after 18 months?

b. How much interest has been earned?

23. Retirement options. At age 25, you start work for a company and are offered two retirement options.

Retirement option 1: When you retire, you will receive a lump sum of $30,000 for each year of service.

Retirement option 2: When you start to work, the company deposits $15,000 into an account that pays a monthly interest rate of 1%, and interest is compounded monthly. When you retire, you get the balance of the account.

Which option is better if you retire at age 65? Which is better if you retire at age 55?

24. Compound interest. Assume that an 18-month CD purchased for $7000 pays an APR of 6% compounded monthly. What is the APY? Would the APY change if the investment were $11,000 for 30 months with the same APR and with monthly compounding?

25. More compound interest. Assume that a 24-month CD purchased for $7000 pays an APY of 4.25% (and of course interest is compounded). How much do you have at maturity?

26. Interest and APY. Assume that a one-year CD purchased for $2000 pays an APR of 8% that is compounded semi-annually. How much is in the account at the end of each compounding period? (Calculate the interest and compound it each period rather than using the compound interest formula.) How much total interest does it earn? What is the APY?

27. More interest and APY. Assume that a one-year CD purchased for $2000 pays an APR of 8% that is compounded quarterly. How much is in the account at the end of each compounding period? (Calculate the interest and compound it each period rather than using the compound interest formula.) How much total interest does it earn? What is the APY?

28. A CD. Suppose you bought a two-year CD for $10,000 at an APR of 3%.

a. What is the APY, assuming monthly compounding?

b. What is the total interest paid on the CD over the two-year period?

29. Some interest and APY calculations. Parts b and c refer to the 2008–2009 rates at First Command Bank shown in Table 4.1.

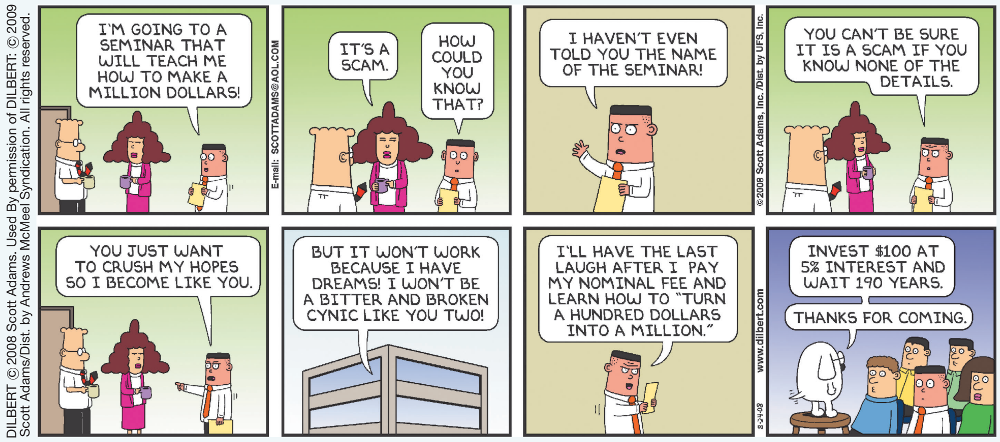
a. Assume that a one-year CD for $5000 pays an APR of 8% that is compounded quarterly. How much total interest does it earn? What is the APY?

b. If you purchased a one-year CD for $150,000 from First Command Bank, how much interest would you have received at maturity? Was compounding taking place? Explain.

c. If you purchased a two-year CD for $150,000 from First Command Bank, the APY (4.05%) was greater than the APR (3.98%) because compounding was taking place. Use the APR to calculate what the APY would be with daily compounding. How does your answer compare to the APY in the table?

30. Interest and APR. Assume that a two-year CD for $4000 pays an APY of 8%. How much interest will it earn? Can you determine the APR?

31. Find the APR. Sue bought a six-month CD for $3000. She said that at maturity it paid $112.50 in interest. Assume that this was simple interest, and determine the APR.



32. Getting rich. The *Dilbert* cartoon shown above jokes about making a million dollars by investing $100 at an APR of 5% and waiting for 190 years. Assuming that interest is compounded annually, how much does this investment really produce?

33. Future value. What is the future value of a 10-year investment of $1000 at an APR of 9% compounded monthly? Explain what your answer means.

34. Present value. What is the present value of an investment that will be worth $2000 at the end of five years? Assume an APR of 6% compounded monthly. Explain what your answer means.

35. Doubling again. You have invested $2500 at an APR of 9%. Use the Rule of 72 to estimate how long it will be until your investment reaches $5000, and how long it will be until your investment reaches $10,000.

36. Getting APR from doubling rate. A friend tells you that her savings account doubled in 12 years. Use the Rule of 72 to estimate what the APR of her account was.

37. Find the doubling time. Consider an investment of $3000 at an APR of 6% compounded monthly. Use the formula that gives the exact doubling time to determine exactly how long it will take for the investment to double. (See the first part of Summary 4.3. Be sure to use the monthly rate for r.) Express your answer in years and months. Compare this result with the estimate obtained from the Rule of 72.

Exercises 38 through 40 are designed to be solved using technology such as calculators or computer spreadsheets.

38. Find the rate. A news article published in 2004 said that the U.S. House of Representatives passed a bill to distribute funds to members of the Western Shoshone tribe. Here is an excerpt from that article:

> The Indian Claims Commission decided the Western Shoshone lost much of their land to gradual encroachment. The tribe was awarded $26 million in 1977. That has grown to about $145 million through compound interest, but the tribe never took the money.

Assume monthly compounding and determine the APR that would give this growth in the award over the 27 years from 1977 to 2004. *Note*: This can also be solved without technology, using algebra.

39. Solve for the APR. Suppose a CD advertises an APY of 8.5%. Assuming that the APY was a result of monthly compounding, solve the equation

$$0.085 = \left(1 + \frac{\text{APR}}{12}\right)^{12} - 1$$

to find the APR. *Note*: This can also be solved without technology, using algebra.

40. Find the compounding period. Suppose a CD advertises an APR of 5.10% and an APY of 5.20%. Solve the equation

$$0.052 = \left(1 + \frac{0.051}{n}\right)^{n} - 1$$

for n to determine how frequently interest is compounded.

Writing About Mathematics

41. Current rates. Look up some current interest rates being paid on CDs by different financial institutions and write a report on your findings.

42. Bond rates. Interest rates on bonds can differ significantly among nations. Look up some current interest rates being paid on bonds by different countries and write a report on your findings.

43. Hamilton. Alexander Hamilton was the first U.S. Treasury secretary. Write a report on Hamilton and his accomplishments.

44. Usury. Look up the meaning of the word *usury* and write a report on how it is viewed in different cultures and religions.

What Do You Think?

45. Present and future value. Explain the meaning of the terms *future value* and *present value*.

46. Simple formula? You and a friend are discussing the return on an account with an initial balance of $100 and an APY of 5%. You want to use the APY balance formula (Formula 4.5). Your friend says that he has a simpler approach: Each year add 5% of $100 or $5 to the balance. Is your friend's approach valid? What term is used to describe the interest calculation employed by your friend? Under what conditions will your friend's approach give a reasonably good solution?

47. Simple versus compound interest. What features would you expect to see if you graph the balance of an account earning simple interest? What features would you expect to see if the account earns compound interest? **Exercise 21 is relevant to this topic.**

48. Which would you choose? You have the option of borrowing money from one source that charges simple interest or from another source that charges the same APR but compounds the interest monthly. Which would you choose, and why? **Exercise 21 is relevant to this topic.**

49. Which would you choose? You have the option of loaning money to one friend who promises to pay simple interest or to another friend who promises to pay the same APR but compound the interest. Which would you choose, and why? **Exercise 21 is relevant to this topic.**

50. Given APR. You are considering two banks. Both banks offer savings accounts with an APR of 2%. What other information would you want to know so that you can choose between the two banks?

51. Doubling the number of compounding periods. If a bank doubles the number of times per year that it compounds interest, does that double the amount you earn in a year on your savings account? Explain.

52. Doubling principal invested. If you double the principal you invest in your savings account, does that double the interest you earn in a year? Explain.

53. APR = APY. A bank tells you that its APR and APY are the same. What does that tell you about compounding?

4.2 Borrowing: How much car can you afford?

TAKE AWAY FROM THIS SECTION
Be able to calculate the monthly payment on a loan.

The following article appeared at the site COLLEGEdata.

IN THE NEWS 4.3

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Paying in Installments

It can be challenging to pay for a whole semester of college in one lump sum. So many colleges offer tuition payment plans that allow you to pay your bill over time. Here's how they work.

Like a No-Interest Loan

Deferred tuition payment plans, also known as tuition installment plans, are a convenience to help you manage college expenses, particularly if you have trouble paying a whole year's tuition at once. Tuition paid in installments must normally be paid off by the end of the relevant academic period, such as a semester or academic year, and most such plans do not charge interest if you pay by check or direct deposit. However, like loans, you must have a good credit history to qualify for them.

The Tuition Deferment Process

Don't show up on the first day of classes expecting to automatically qualify for such a plan. You should discuss any payment plans beforehand with your college. The plan may be handled by a private company or by the college. In either case you will usually pay a relatively small service fee and, if appropriate, late fees. The first payment of an installment plan is normally due at registration and may be the largest. If you enroll at a college that does not offer such a plan, its financial aid office may be able to refer you to a private commercial tuition management company that does.

(Continued)

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Paying in Installments (*Continued*)

Beware Hidden Costs

Installment plans can end up costing you more in the long run if you drop out or transfer to another college. Because the college expects costs to go up each year, payments in the first two years of college may be larger than they would be otherwise. An additional cost may also result if you pay by credit card. Some plans charge a fee for this service, as high as nearly three percent. To determine your college's policy, check with its bursar's office.

Tuition payment plans are not for everybody, but it's good to know they're available if you need them. It's one more way colleges try to help you pay for your education.

Installment payments are a part of virtually everyone's life. The goal of this section is to explain how such payments are calculated and to see the implications of installment plans.

Installment loans and the monthly payment

When we borrow money to buy a car or a house, the lending institution typically requires that we pay back the loan plus interest by making the same payment each month for a certain number of months. Note that interest rates for mortgages and various other loans change over time. Thus, in many of the examples and exercises in this section we use interest rates that may not be current, just as we did with CD interest rates in Section 4.1.

KEY CONCEPT

With an installment loan you borrow money for a fixed period of time, called the term of the loan, and you make regular payments (usually monthly) to pay off the loan plus interest accumulated during that time.

To see how the monthly payment is calculated, we consider a simple example. Suppose you need \$100 to buy a calculator but don't have the cash available. Your sister is willing, however, to lend you the money at a rate of 5% per month, provided you repay it with equal payments over the next two months. (By the way, this is an astronomical APR of 60%.)

How much must you pay each month? Because the loan is for only two months, you need to pay at least half of the \$100, or \$50, each of the two months. But that doesn't account for the interest. If we were to pay off the loan in a lump sum at the end of the two-month term, the interest on the account would be calculated in the same way as for a savings account that pays 5% per month. Using the compound interest formula (Formula 4.3 from Section 4.1), we find

$$\begin{aligned}\text{Balance owed after 2 months } &= \text{Principal} \times (1 + r)^t \\ &= \$100 \times 1.05^2 = \$110.25\end{aligned}$$

This would be two monthly payments of approximately \$55.13. This amount overestimates your monthly payment, however: in the second payment, you shouldn't have to pay interest on the amount you have already repaid.

We will see shortly that the correct payment is \$53.78 for each of the two months. This is not an obvious amount, but it is reasonable—it lies between the two extremes of \$50 and \$55.13.

When we take out an installment loan, the amount of the payment depends on three things: the amount of money we borrow (sometimes called the *principal*), the interest rate (or APR), and the term of the loan.

Each payment reduces the balance owed, but at the same time interest is accruing on the outstanding balance. This makes the calculation of the monthly payment fairly complicated, as you might surmise from the following formula.[3]

FORMULA 4.6 Monthly Payment Formula

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

Here, t is the term in months and $r = \text{APR}/12$ is the monthly interest rate as a decimal.

Alternatively, we can write the formula as

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times \dfrac{\text{APR}}{12}\left(1 + \dfrac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1 + \dfrac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

where y is the term in years.

A derivation of this formula is presented at the end of this section in Algebraic Spotlight 4.2 and Algebraic Spotlight 4.3.

To give a sense of these monthly payments, here are two examples that might arise when getting a mortgage:

- If you borrowed \$100,000 at 6% APR for 30 years, your monthly payment would be approximately \$600.
- If you borrowed \$100,000 at 3% APR for 15 years, your monthly payment would be approximately \$690.

If you want to borrow money, the monthly payment formula allows you to determine in advance whether you can afford that car or home you want to buy. The formula also lets you check the accuracy of any figure that a potential lender quotes.

Let's return to the earlier example of a \$100 loan to buy a calculator. We have a monthly rate of 5%. Expressed as a decimal, 5% is 0.05, so $r = 0.05$ and $1 + r = 1.05$. Because we pay off the loan in two months, we use $t = 2$ in the monthly payment formula (Formula 4.6):

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$100 \times 0.05 \times 1.05^2}{(1.05^2 - 1)} \\ &= \$53.78\end{aligned}$$

[3] *This formula is built in as a feature on some hand-held calculators and most computer spreadsheets. Loan calculators are also available on the Web.*

EXAMPLE 4.9 Using the monthly payment formula: College loan

You need to borrow \$5000 so you can attend college next fall. You get the loan at an APR of 6% to be paid off in monthly installments over three years.[4]

a. Calculate your monthly payment.

b. What is the total of all payments?

c. How much interest was paid in all?

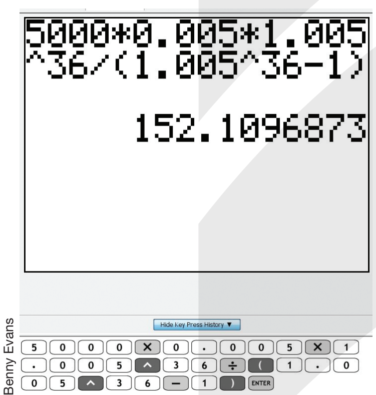
SOLUTION

a. The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.06}{12} = 0.005$$

This gives $1 + r = 1.005$. We want to pay off the loan in three years or 36 months, so we use a term of $t = 36$ in the monthly payment formula:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$5000 \times 0.005 \times 1.005^{36}}{(1.005^{36} - 1)} \\ &= \$152.11\end{aligned}$$



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b. There are 36 payments of \$152.11 each, so the total of all payments is

$$36 \times \$152.11 = \$5475.96$$

c. Because the total payments are \$5475.96, of which \$5000 was the amount borrowed, the difference,

$$\$5475.96 - 5000 = \$475.96,$$

is the amount of interest paid over the life of the loan.

TRY IT YOURSELF 4.9

You borrow \$8000 at an APR of 9% to be paid off in monthly installments over four years. Calculate your monthly payment.

The answer is provided at the end of this section.

Suppose you can afford a certain monthly payment and you'd like to know how much you can borrow to stay within that budget. The monthly payment formula can be rearranged to answer that question.

[4] *In April 2021, federally subsidized student loans were around 2.75% APR, and some private loans were advertised at about 4.25% APR.*

FORMULA 4.7 Companion to the Monthly Payment Formula (Formula for Amount Borrowed)

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

where t is the term of the loan in months.

Alternatively, we can write the formula as

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

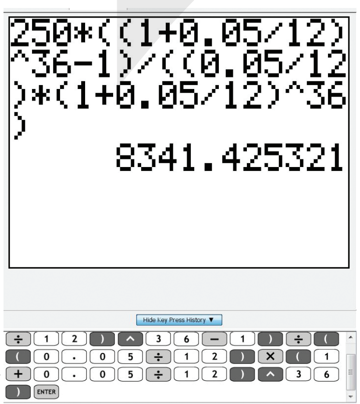
where y is the term of the loan in years.

EXAMPLE 4.10 Computing how much I can borrow: Buying a car

We can afford to make payments of \$250 per month for three years. Our car dealer is offering us a loan at an APR of 5%. What price automobile should we be shopping for?

© Westend61 GmbH/Alamy

Before you shop for a car, know how much car you can afford.



Benny Evans

SOLUTION

The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{0.05}{12}$$

To four decimal places, this is 0.0042, but for better accuracy, we won't round this number. Now, three years is 36 months, so we use $t = 36$ in the companion payment formula (Formula 4.7):

$$\begin{aligned} \text{Amount borrowed} &= \frac{\text{Monthly payment} \times ((1+r)^t - 1)}{(r \times (1+r)^t)} \\ &= \frac{\$250 \times ((1+0.05/12)^{36} - 1)}{((0.05/12) \times (1+0.05/12)^{36})} \\ &= \$8341.43 \end{aligned}$$

We should shop for cars that cost \$8341.43 or less.

TRY IT YOURSELF 4.10

We can afford to make payments of \$300 per month for four years. We can get a loan at an APR of 4%. How much money can we afford to borrow?

The answer is provided at the end of this section.

SUMMARY 4.4
Monthly Payments

In parts 1 and 2, the monthly rate r is the APR in decimal form divided by 12, and t is the term in months.

1. The monthly payment is

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

2. A companion formula gives the amount borrowed in terms of the monthly payment:

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

3. Alternatively, we can write these formulas in term of y, the term of the loan in years:

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times \frac{\text{APR}}{12}\left(1 + \frac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

and

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

4. These formulas are sensitive to round-off error, so it is best to do calculations all at once, keeping all the decimal places rather than doing parts of a computation and entering the rounded numbers.

EXAMPLE 4.11 Calculating monthly payment and amount borrowed: A new car

Suppose we need to borrow \$15,000 at an APR of 9% to buy a new car.

a. What will the monthly payment be if we borrow the money for $3\frac{1}{2}$ years? How much interest will we have paid by the end of the loan?

b. We find that we cannot afford the \$15,000 car because we can only afford a monthly payment of \$300. What price car can we shop for if the dealer offers a loan at a 9% APR for a term of $3\frac{1}{2}$ years?

SOLUTION

a. The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.09}{12} = 0.0075$$

We are paying off the loan in $3\frac{1}{2}$ years, so $t = 3.5 \times 12 = 42$ months. Therefore, by the monthly payment formula (Formula 4.6),

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

$$= \frac{\$15{,}000 \times 0.0075 \times 1.0075^{42}}{(1.0075^{42} - 1)}$$

$$= \$417.67$$

Now let's find the amount of interest paid. We will make 42 payments of \$417.67 for a total of $42 \times \$417.67 = \$17{,}542.14$. Because the amount we borrowed is \$15,000, this means that the total amount of interest paid is $\$17{,}542.14 - \$15{,}000 = \$2542.14$.

b. The monthly interest rate as a decimal is $r = 0.0075$, and there are still 42 payments. We know that the monthly payment we can afford is \$300. We use the companion formula (Formula 4.7) to find the amount we can borrow on this budget:

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

$$= \frac{\$300 \times (1.0075^{42} - 1)}{(0.0075 \times 1.0075^{42})}$$

$$= \$10{,}774.11$$

This means that we can afford to shop for a car that costs no more than \$10,774.11.

The next example shows how saving compares with borrowing.

EXAMPLE 4.12 Comparing saving versus borrowing: A loan and a CD

a. Suppose we borrow \$5000 for one year at an APR of 7.5%. What will the monthly payment be? How much interest will we have paid by the end of the year?

b. Suppose we were able to buy a one-year \$5000 CD at an APR of 7.5% compounded monthly. How much interest will be paid at the end of the year?

c. In part a, the financial institution loaned us \$5000 for one year, but in part b we loaned the financial institution \$5000 for one year. What is the difference in the amount of interest paid? Explain why the amounts are different.

SOLUTION

a. In this case, the principal is \$5000, the monthly interest rate r as a decimal is $0.075/12 = 0.00625$, and the number t of payments is 12. We use the monthly payment formula (Formula 4.6):

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

$$= \frac{\$5000 \times 0.00625 \times 1.00625^{12}}{(1.00625^{12} - 1)}$$

$$= \$433.79$$

We will make 12 payments of \$433.79 for a total of $12 \times \$433.79 = \$5205.48$. Because the amount we borrowed is \$5000, the total amount of interest paid is \$205.48.

b. To calculate the interest earned on the CD, we use the compound interest formula (Formula 4.3 from the preceding section). The monthly rate is the same as in part a. Therefore,

$$\begin{aligned}\text{Balance} &= \text{Principal} \times (1 + r)^t \\ &= \$5000 \times 1.00625^{12} \\ &= \$5388.16\end{aligned}$$

This means that the total amount of interest we earned is \$388.16.

c. The interest earned on the \$5000 CD is \$182.68 more than the interest paid on the \$5000 loan in part a.

Here is the explanation for this difference: When we save money, the financial institution credits our account with an interest payment—in this case, every month. We continue to earn interest on the full \$5000 and on those interest payments for the entire year. When we borrow money, however, we repay the loan monthly, thus decreasing the balance owed. We are being charged interest only on the balance owed, not on the full \$5000 we borrowed.

One common piece of financial advice is to reduce the amount you borrow by making a *down payment*. Exercise 22 explores the effect of making a down payment on the monthly payment and the total interest paid.

Estimating payments for short-term loans

The formula for the monthly payment is complicated, and it's easy to make a mistake in the calculation. Is there a simple way to give an estimate for the monthly payment? There are, in fact, a couple of ways to do this. We give one of them here and another when we look at home mortgages.

One obvious estimate for a monthly payment is to divide the loan amount by the term (in months) of the loan. *This would be our monthly payment if no interest were charged*. For a loan with a relatively short term and not too large an interest rate, this gives a rough lower estimate for the monthly payment.[5]

RULE OF THUMB 4.2 Monthly Payments for Short-Term Loans

For all loans, the monthly payment is *at least* the amount we would pay each month if no interest were charged, which is the amount of the loan divided by the term (in months) of the loan. This would be the payment if the APR were 0%. It's a rough estimate of the monthly payment for a short-term loan if the APR is not large.

EXAMPLE 4.13 Estimating monthly payment: Can we afford it?

The largest monthly payment we can afford is \$800. Can we afford to borrow a principal of \$20,000 with a term of 24 months?

SOLUTION

The rule of thumb says that the monthly payment is at least $\$20{,}000/24 = \$833.33$. This is more than \$800, so we can't afford this loan.

---

[5] *If the term is at most five years and the APR is less than 7.5%, the actual payment is within 20% of the ratio. If the term is at most two years and the APR is less than 9%, the actual payment is within 10% of the ratio.*

TRY IT YOURSELF 4.13

The largest monthly payment we can afford is \$450. Can we afford to borrow a principal of \$18,000 with a term of 36 months?

The answer is provided at the end of this section.

For the loan in the preceding example, our rule of thumb says that the monthly payment is *at least* \$833.33. Remember that this amount does not include the interest payments. If the loan has an APR of 6.6%, for example, the actual payment is \$891.83.

Once again, a rule of thumb gives an estimate—not the exact answer. It can at least tell us quickly whether we should be shopping on the BMW car lot.

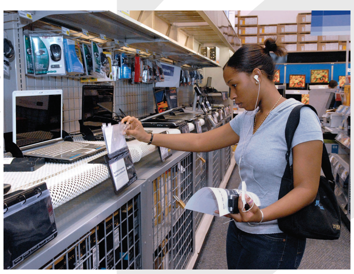
Amortization tables and equity

When you make payments on an installment loan, part of each payment goes toward interest, and part goes toward reducing the balance owed. An *amortization table* is a running tally of payments made and the outstanding balance owed.

KEY CONCEPT

An amortization table or amortization schedule for an installment loan shows for each payment made the amount applied to interest, the amount applied to the balance owed, and the outstanding balance.

EXAMPLE 4.14 Making an amortization table: Buying a computer


Jim Stern/Bloomberg/Getty Images

Suppose we borrow \$1000 at 12% APR to buy a computer. We pay off the loan in 12 monthly payments. Make an amortization table showing payments over the first six months.

SOLUTION

The monthly rate is $12\%/12 = 1\%$. As a decimal, this is $r = 0.01$. The monthly payment formula with $t = 12$ gives

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

$$= \frac{\$1000 \times 0.01 \times 1.01^{12}}{(1.01^{12} - 1)}$$

$$= \$88.85$$

Because the monthly rate is 1%, each month we pay 1% of the outstanding balance in interest. When we make our first payment, the outstanding balance is \$1000, so we pay 1% of \$1000, or \$10.00, in interest. Thus, \$10 of our \$88.85 goes toward interest, and the remainder, \$78.85, goes toward the outstanding balance. So after the first payment, we owe:

$$\text{Balance owed after 1 payment} = \$1000.00 - \$78.85 = \$921.15$$

When we make a second payment, the outstanding balance is \$921.15. We pay 1% of \$921.15 or \$9.21 in interest, so $\$88.85 - \$9.21 = \$79.64$ goes toward the balance due. This gives the balance owed after the second payment:

$$\text{Balance owed after 2 payments} = \$921.15 - \$79.64 = \$841.51$$

If we continue in this way, we get the following table:

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				\$1000.00
1	\$88.85	1% of \$1000.00 = \$10.00	\$78.85	\$921.15
2	\$88.85	1% of \$921.15 = \$9.21	\$79.64	\$841.51
3	\$88.85	1% of \$841.51 = \$8.42	\$80.43	\$761.08
4	\$88.85	1% of \$761.08 = \$7.61	\$81.24	\$679.84
5	\$88.85	1% of \$679.84 = \$6.80	\$82.05	\$597.79
6	\$88.85	1% of \$597.79 = \$5.98	\$82.87	\$514.92

TRY IT YOURSELF 4.14

Suppose we borrow \$1000 at 30% APR and pay it off in 24 monthly payments. Make an amortization table showing payments over the first three months.

The answer is provided at the end of this section.

The loan in Example 4.14 is used to buy a computer. The amount you have paid toward the actual cost of the computer (the principal) at a given time is referred to as your *equity* in the computer. For example, the preceding table tells us that after four payments we still owe \$679.84. This means we have paid a total of \$1000.00 − \$679.84 = \$320.16 toward the principal. That is our equity in the computer.

KEY CONCEPT

If you borrow money to pay for an item, your equity in that item at a given time is the part of the principal you have paid.

EXAMPLE 4.15 Calculating equity: Buying land

You borrow \$150,000 at an APR of 6% to purchase a plot of land. You pay off the loan in monthly payments over 10 years.

a. Find the monthly payment.

b. Complete the four-month amortization table below.

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				\$150,000.00
1				
2				
3				
4				

c. What is your equity in the land after four payments?

SOLUTION

a. The monthly rate is APR/12 = 6%/12 = 0.5%. As a decimal, this is $r = 0.005$. We use the monthly payment formula with $t = 10 \times 12 = 120$ months:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$150{,}000 \times 0.005 \times 1.005^{120}}{(1.005^{120} - 1)} \\ &= \$1665.31\end{aligned}$$

b. For the first month, the interest we pay is 0.5% of the outstanding balance of $150,000:

$$\text{First month interest} = \$150{,}000 \times 0.005 = \$750$$

Now $750 of the $1665.31 payment goes to interest and the remainder, $1665.31 − $750 = $915.31, goes toward reducing the principal. The balance owed after one month is

$$\text{Balance owed after 1 month} = \$150{,}000 - \$915.31 = \$149{,}084.69$$

This gives the first row of the table. The completed table is shown below.

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				$150,000.00
1	$1665.31	0.5% of $150,000.00 = $750.00	$915.31	$149,084.69
2	$1665.31	0.5% of $149,084.69 = $745.42	$919.89	$148,164.80
3	$1665.31	0.5% of $148,164.80 = $740.82	$924.49	$147,240.31
4	$1665.31	0.5% of $147,240.31 = $736.20	$929.11	$146,311.20

c. The table from part b tells us that after four payments we still owe $146,311.20. So our equity is

$$\text{Equity after 4 months} = \$150,000 - \$146,311.20 = \$3688.80$$

The graph in **Figure 4.6** shows the percentage of each payment from Example 4.15 that goes toward interest, and **Figure 4.7** shows how equity is built. Note that in the early months, a large percentage of the payment goes toward interest. For long-term loans, an even larger percentage of the payment goes toward interest early on. This means that equity is built slowly at first. The rate of growth of equity increases over the life of the loan. Note in Figure 4.7 that when you have made half of the payments (60 payments), you have built an equity of just over $60,000—much less than half of the purchase price.

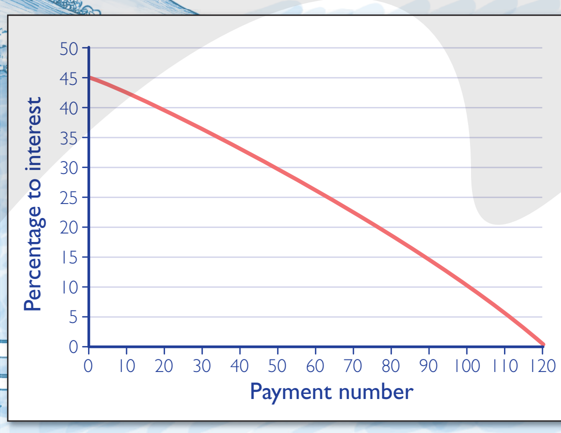


FIGURE 4.6 Percentage of payment that goes to interest: 10-year loan.

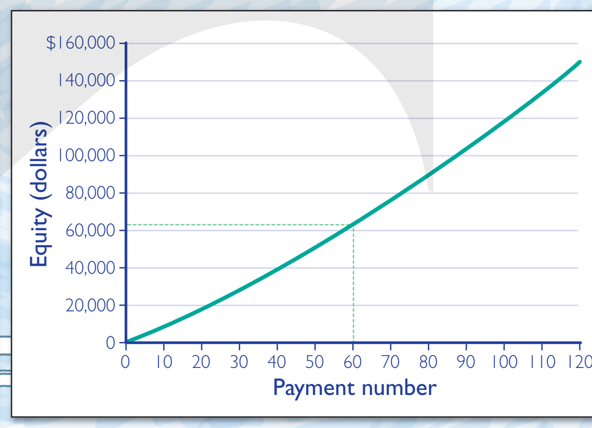


FIGURE 4.7 Equity built: 10-year loan.

Home mortgages

Figure 4.8 shows that, like other interest rates, mortgage rates have fluctuated over the years, and have recently been at historic lows. As of April 2021, the average rate for a 30-year mortgage was 3.25%.

FIGURE 4.8 A graph with data from Freddie Mac of 30-year fixed rate mortgage averages in the United States, from 1970 to the present.

A home mortgage is a loan for the purchase of a home. It is very common for a mortgage to last as long as 30 years. The early mortgage payments go almost entirely toward interest, with a small part going to reduce the principal. You can see an example of this in the lower right corner of **Figure 4.9**. As a consequence, your home equity grows very slowly. For a 30-year mortgage of $150,000 at an APR of 6%, **Figure 4.10** shows the percentage of each payment that goes to interest, and **Figure 4.11** shows the equity built. Your home equity is very important because it tells you how much money you can actually keep if you sell your house, and it can also be used as collateral to borrow money.

Springside Mortgage
Customer Service: 1-800-555-1234
www.springsidemortgage.com

Jordan and Dana Smith
4700 Jones Drive
Memphis, TN 38109

Mortgage Statement
Statement Date: 3/20/2021

Account Number	1234567
Payment Due Date	4/1/2021
Amount Due	**$2,079.71**

If payment is received after 4/15/21, $160 late fee will be charged.

Account Information	
Outstanding Principal	$264,776.43
Interest Rate (Until October 2021)	4.75%
Prepayment Penalty	$3,500.00

Explanation of Amount Due	
Principal	$386.46
Interest	$1,048.07
Escrow (for Taxes and Insurance)	$235.18
Regular Monthly Payment	**$1,669.71**
Total Fees Charged	$410.00
Total Amount Due	**$2,079.71**

FIGURE 4.9 A typical mortgage statement. Note the small amount of the payment going to reduce the principal.

FIGURE 4.10 Percentage of payment that goes to interest: 30-year mortgage.

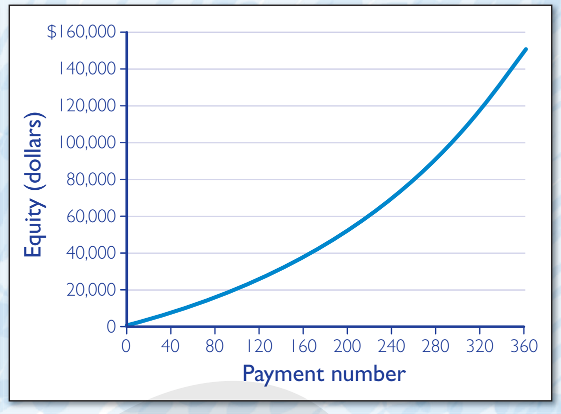


FIGURE 4.11 Equity built: 30-year mortgage.

EXAMPLE 4.16 Computing interest: 30-year mortgage

Your neighbor took out a 30-year mortgage for \$300,000 at a time when the APR was 9%. She says that she will wind up paying more in interest than for the home (that is, the principal). Is that true?

SOLUTION

We first need to find the monthly payment. The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.09}{12} = 0.0075$$

Because the loan is for 30 years, we use $t = 30 \times 12 = 360$ months in the monthly payment formula:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$300{,}000 \times 0.0075 \times 1.0075^{360}}{(1.0075^{360} - 1)} \\ &= \$2413.87\end{aligned}$$

She will make 360 payments of \$2413.87, for a total of

$$\text{Total amount paid} = 360 \times \$2413.87 = \$868{,}993.20$$

The interest paid is the excess over \$300,000, or \$568,993.20. Your neighbor paid almost twice as much in interest as she did for the home.

TRY IT YOURSELF 4.16

Find the interest paid on a 25-year mortgage of \$450,000 at an APR of 4.3%.

The answer is provided at the end of this section.

Now we see the effect of varying the term of the mortgage on the monthly payment.

EXAMPLE 4.17 Determining monthly payment and term: Choices for mortgages

You need to secure a loan of $250,000 to purchase a home. Your lending institution offers you three options:

Option 1: A 30-year mortgage at 8.4% APR.

Option 2: A 20-year mortgage at 7.2% APR.

Option 3: A 30-year mortgage at 7.2% APR, including a fee of 4 *loan points*. *Note*: “Points” are a fee you pay for the loan in return for a decrease in the interest rate. In this case, a fee of 4 points means you pay 4% of the loan, or $10,000. One way to do this is to borrow the fee from the bank by just adding the $10,000 to the amount you borrow. The bank keeps the $10,000 and the other $250,000 goes to buy the home.

Determine the monthly payment for each of these options.

SOLUTION

Option 1: The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.084}{12} = 0.007$$

We use the monthly payment formula with $t = 360$ months:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$250{,}000 \times 0.007 \times 1.007^{360}}{(1.007^{360} - 1)} \\ &= \$1904.59\end{aligned}$$

Option 2: The APR for a 20-year loan is lower. (It is common for loans with shorter terms to have lower interest rates.) An APR of 7.2% is a monthly rate of $r = 0.072/12 = 0.006$ as a decimal. We use the monthly payment formula with $t = 20 \times 12 = 240$ months:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$250{,}000 \times 0.006 \times 1.006^{240}}{(1.006^{240} - 1)} \\ &= \$1968.37\end{aligned}$$

The monthly payment is about $60 higher than for the 30-year mortgage, but you pay the loan off in 20 years rather than 30 years.

Option 3: Adding 4% to the amount we borrow gives a new loan amount of $260,000. As with Option 2, the APR of 7.2% gives a monthly rate of $r = 0.006$. The term of 30 years means that we put $t = 360$ months into the monthly payment formula:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$260{,}000 \times 0.006 \times 1.006^{360}}{(1.006^{360} - 1)} \\ &= \$1764.85\end{aligned}$$

Getting the lower interest rate makes a big difference in the monthly payment, even with the 4% added to the original loan balance. This is clearly a better choice than Option 1 if we consider only the monthly payment. Option 3 does require borrowing more money than Option 1 and could have negative consequences if you need to sell the home early. Further, comparing the amount of interest paid shows that Option 2 is the best choice from that point of view—if you can afford the monthly payment.

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"As an alternative to the traditional 30-year mortgage, we also offer an interest-only mortgage, balloon mortgage, reverse mortgage, upside down mortgage, inside out mortgage, loop-de-loop mortgage, and the spinning double axel mortgage with a triple lutz."

Adjustable-rate mortgages

In the summer of 2007, a credit crisis involving home mortgages had dramatic effects on many people and ultimately on the global economy. Several major financial institutions collapsed in the autumn of 2008, and more than 3 million people lost their homes through foreclosure.[6] There were many causes of the crisis, but one prominent cause was the increase in risky mortgage lending, a large percentage of which involved *adjustable-rate mortgages* or ARMs.

KEY CONCEPT

A fixed-rate mortgage keeps the same interest rate over the life of the loan. In the case of an adjustable-rate mortgage or ARM, the interest rate may vary over the life of the loan. The rate is often tied to the prime interest rate, which is the rate banks must pay to borrow money.

Kimberly White/Bloomberg via Getty Images

One advantage of an ARM is that the initial rate is often lower than the rate for a comparable fixed-rate mortgage. One disadvantage of an ARM is that a rising prime interest rate may cause significant increases in the monthly payment. That is what happened in 2007–2008. When housing prices began to fall and interest rates for adjustable-rate mortgages began to grow, people had higher monthly payments while the value of their homes fell. Thus, mortgage delinquencies increased. This contributed to financial markets becoming concerned about the soundness of U.S. credit, which led to slowing economic growth in the United States and Europe.

EXAMPLE 4.18 Comparing monthly payments: Fixed-rate and adjustable-rate mortgages

We want to borrow \$200,000 for a 30-year home mortgage. We have found an APR of 6.6% for a fixed-rate mortgage and an APR of 6% for an ARM. Compare the initial monthly payments for these loans.

[6] http://usatoday30.usatoday.com/money/economy/housing/2009-01-14-foreclosure-record-filings_N.htm

SOLUTION

Both loans have a principal of \$200,000 and a term of $t = 360$ months. For the fixed-rate mortgage, the monthly rate in decimal form is $r = 0.066/12 = 0.0055$. The monthly payment formula gives

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$200{,}000 \times 0.0055 \times 1.0055^{360}}{(1.0055^{360} - 1)} \\ &= \$1277.32\end{aligned}$$

For the ARM, the initial monthly rate in decimal form is $r = 0.06/12 = 0.005$. The monthly payment formula gives

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$200{,}000 \times 0.005 \times 1.005^{360}}{(1.005^{360} - 1)} \\ &= \$1199.10\end{aligned}$$

The initial monthly payment for the ARM is almost \$80 less than the payment for the fixed-rate mortgage—but the payment for the ARM could change at any time.

TRY IT YOURSELF 4.18

We want to borrow \$150,000 for a 30-year home mortgage. We have found an APR of 6% for a fixed-rate mortgage and an APR of 5.7% for an ARM. Compare the monthly payments for these loans.

The answer is provided at the end of this section.

Example 4.18 illustrates why an ARM may seem attractive. The following example illustrates one potential danger of an ARM. It is typical of what happened in 2007 to many families who took out loans in 2006 when interest rates fell to historic lows.

EXAMPLE 4.19 Using an ARM: Effect of increasing rates

Suppose a family has an annual income of \$60,000. Assume that this family secures a 30-year ARM for \$250,000 at an initial APR of 4.5%.

a. Find the family's monthly payment and the percentage of its monthly income used for the mortgage payment.

b. Now suppose that after one year, the rate adjusts to 6%. Find the family's new monthly payment, the increase in the family's monthly payment, and the percentage of its monthly income now required for the mortgage payment.

SOLUTION

a. The monthly rate as a decimal is $0.045/12 = 0.00375$, so with $t = 360$ the monthly payment on the family's home is

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$250{,}000 \times 0.00375 \times 1.00375^{360}}{(1.00375^{360} - 1)} \\ &= \$1266.71\end{aligned}$$

An annual income of \$60,000 is a monthly income of \$5000. To find the percentage of the family's monthly income used for the mortgage payment, we calculate 1266.71/5000. The result is about 0.25, so the family is paying 25% of its monthly income for the mortgage payment.

b. The APR has increased to 6%, so $r = 0.06/12 = 0.005$. To find the new monthly payment, we use a loan period of 29 years or $29 \times 12 = 348$ months. For the principal, we use the balance owed to the bank after one year of payments. We noted earlier in this section that homeowners build negligible equity in the first year of payments, so we will get a very good estimate of the new monthly payment if we use \$250,000 as the balance owed to the bank. The formula gives

$$\frac{\$250{,}000 \times 0.005 \times 1.005^{348}}{(1.005^{348} - 1)} = \$1517.51$$

Therefore, the new monthly payment is \$1517.51. The difference, \$1517.51–\$1266.71 = \$250.80, is the increase in the family's monthly payment. To find the percentage of their monthly income required for the payment, we divide \$1517.51 by the monthly income of \$5000. The result is approximately 0.30, or 30%.

In the example, the extra burden on family finances caused by the increase in the interest rate could lead to serious problems. Further rate adjustments could easily result in the loss of the family home.

Estimating payments on long-term loans

The payment on a long-term loan is *at least* as large as the monthly interest on the original balance. This is a pretty good estimate for mortgages in times of moderate-to-high interest rates.[7]

RULE OF THUMB 4.3 Monthly Payments for Long-Term Loans

For all loans, the monthly payment is *at least* as large as the principal times the monthly interest rate as a decimal. This is a fairly good estimate for a long-term loan with a moderate or high interest rate.

Let's see what this rule of thumb would estimate for the payment on a mortgage of \$100,000 at an APR of 7.2%. The monthly rate is 7.2%/12 = 0.6%. The monthly interest on the original balance is 0.6% of \$100,000:

$$\text{Monthly payment estimate} = \$100{,}000 \times 0.006 = \$600$$

If this is a 30-year mortgage, the monthly payment formula gives the value \$678.79 for the actual payment. This is about 13% higher than the estimate.

We should note that most home mortgage payments also include taxes and insurance. These vary from location to location but can be significant.[8] For many, a home mortgage is the most significant investment they will ever make. It is crucial to understand clearly both the benefits and the costs of such an investment.

[7] *If the term is at least 30 years and the APR is at least 6%, then the actual payment is within 20% of the estimate. If the term is at least 30 years and the APR is greater than 8%, then the actual payment is within 10% of the estimate.*

[8] *See Exercise 31 (on affordability).*

ALGEBRAIC SPOTLIGHT 4.2 Derivation of the Monthly Payment Formula, Part I

First we derive a formula for the balance still owed on an installment loan after a given number of payments. Suppose we borrow B_0 dollars and make monthly payments of M dollars. Suppose further that interest accrues on the balance at a monthly rate of r as a decimal. Let B_n denote the account balance (in dollars) after n months. Our goal is to derive the formula

$$B_n = B_0(1+r)^n - M\frac{(1+r)^n - 1}{r}$$

Each month we find the new balance B_{n+1} from the old balance B_n by first adding the interest accrued rB_n and then subtracting the payment M. As a formula, this is

$$B_{n+1} = B_n + rB_n - M = B_n(1+r) - M$$

If we put $R = 1 + r$, this formula can be written more compactly as

$$B_{n+1} = B_n R - M$$

Repeated application of this formula gives

$$B_n = B_0 R^n - M(1 + R + R^2 + \cdots + R^{n-1})$$

To finish, we use the *geometric sum formula*, which tells us that

$$1 + R + R^2 + \cdots + R^{n-1} = \frac{R^n - 1}{R - 1}$$

This gives

$$B_n = B_0 R^n - M\frac{R^n - 1}{R - 1}$$

Finally, we recall that $R = 1 + r$ and obtain the desired formula

$$B_n = B_0(1+r)^n - M\frac{(1+r)^n - 1}{r}$$

ALGEBRAIC SPOTLIGHT 4.3 Derivation of the Monthly Payment Formula, Part II

Now we use the account balance formula to derive the monthly payment formula. Suppose we borrow B_0 dollars at a monthly interest rate of r as a decimal and we want to pay off the loan in t monthly payments. That is, we want the balance to be 0 after t payments. Using the account balance formula we derived in Algebraic Spotlight 4.2, we want to find the monthly payment M that makes $B_t = 0$. That is, we need to solve the equation

$$0 = B_t = B_0(1+r)^t - M\frac{(1+r)^t - 1}{r}$$

for M. Now

$$0 = B_0(1+r)^t - M\frac{(1+r)^t - 1}{r}$$

$$M\frac{(1+r)^t - 1}{r} = B_0(1+r)^t$$

$$M = \frac{B_0 r(1+r)^t}{((1+r)^t - 1)}$$

Because B_0 is the amount borrowed and t is the term, this is the monthly payment formula we stated earlier.

WHAT DO YOU THINK?

High interest rate: You are considering taking out a loan but are concerned about the high interest rate. Should you be more concerned if you anticipate repaying the loan over 2 years or 20 years? Explain your reasoning.

Questions of this sort may invite many correct responses. We present one possibility: Assuming that interest is compounded, a high interest rate has a greater effect on a long-term loan than on a short-term loan. That is because of the exponential nature of compound interest, where interest is charged on accrued interest. The shape of an exponential graph shows that growth over the long term is very rapid. For a loan, growth is offset by payments, but the influence of high interest nonetheless causes a greater burden over long terms.

Further questions of this type may be found in the *What Do You Think?* section of exercises.

Try It Yourself answers

Try It Yourself 4.9: Using the monthly payment formula: College loan $199.08.

Try It Yourself 4.10: Computing how much I can borrow: Buying a car $13,286.65.

Try It Yourself 4.13: Estimating monthly payment: Can we afford it? No: The monthly payment is at least $500.

Try It Yourself 4.14: Making an amortization table: Buying a computer

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				$1000.00
1	$55.91	2.5% of $1000.00 = $25.00	$30.91	$969.09
2	$55.91	2.5% of $969.09 = $24.23	$31.68	$937.41
3	$55.91	2.5% of $937.41 = $23.44	$32.47	$904.94

Try It Yourself 4.16: Computing how much interest: 25-year mortgage $285,132.00

Try It Yourself 4.18: Comparing monthly payments: Fixed-rate and adjustable-rate mortgages The monthly payment of $870.60 for the ARM is almost $30 less than the payment of $899.33 for the fixed-rate mortgage.

Exercise Set 4.2

Test Your Understanding

1. For an installment loan with monthly compounding, list the factors that determine your monthly payment.

2. True or false: For a loan, your monthly payment is at least the amount borrowed divided by the number of months required to pay off the loan.

3. True or false: If you borrow to buy a car, your monthly payment is at least as large as the amount borrowed times the monthly interest rate (as a decimal).

4. If you borrow to buy a car, your equity in the car is ________.

5. True or false: For a long-term loan such as a mortgage, the total interest you pay may exceed the original principal.

6. The abbreviation ARM stands for ________.

7. True or false: An adjustable-rate mortgage may be risky because your interest rates could go up.

8. An amortization table for a loan shows you: **a.** the amount of interest you have paid each month **b.** the part of the principal you have paid each month **c.** your monthly payment **d.** all of these.

Problems

Rounding in the calculation of monthly interest rates is discouraged. Such rounding can lead to answers different from those presented here. For long-term loans, the differences may be pronounced.

9. A friend said that he borrowed $1200 for one year and his monthly payments were $90. Explain why you are certain that your friend is mistaken.

10. Bill said that he wants to borrow $6000 to buy a used car and pay off the loan in five years, but he says he can't afford more than $100 per month. Can Bill afford this car? Explain.

11. Estimating monthly payment. Use Rule of Thumb 4.2 to estimate the monthly payment on a loan of $5000 borrowed over a three-year period.

12. Estimating monthly payment. Use Rule of Thumb 4.3 to estimate the monthly payment on a loan of $200,000 at an APR of 6% over a period of 25 years.

13. Car payment. To buy a car, you borrow $20,000 with a term of five years at an APR of 6%. What is your monthly payment? How much total interest is paid?

14. Truck payment. You borrow $18,000 with a term of four years at an APR of 5% to buy a truck. What is your monthly payment? How much total interest is paid?

15. Estimating the payment. You borrow $25,000 with a term of two years at an APR of 5%. Use Rule of Thumb 4.2 to estimate your monthly payment, and compare this estimate with what the monthly payment formula gives.

16. Estimating a mortgage. Several years ago Bill got a home mortgage of $110,000 with a term of 30 years at an APR of 9%. Use Rule of Thumb 4.3 to estimate his monthly payment, and compare this estimate with what the monthly payment formula gives.

17. Affording a car. You can get a car loan with a term of three years at an APR of 5%. If you can afford a monthly payment of $450, how much can you borrow?

18. Affording a home. You find that the going rate for a home mortgage with a term of 30 years is 6.5% APR. The lending agency says that based on your income, your monthly payment can be at most $750. How much can you borrow?

19. No interest. A car dealer offers you a loan with no interest charged for a term of two years. If you need to borrow $18,000, what will your monthly payment be? Which rule of thumb is relevant here?

20. Interest paid. Assume that you take out a $2000 loan for 30 months at 8.5% APR.

a. What is the monthly payment?

b. How much of the first month's payment is interest?

c. What percentage of the first month's payment is interest? (Round your answer to two decimal places as a percentage.)

d. How much total interest will you have paid at the end of the 30 months?

21. More saving and borrowing. In this exercise, we compare saving and borrowing as in Example 4.12.

a. Suppose you borrow $10,000 for two years at an APR of 8.75%. What will your monthly payment be? How much interest will you have paid by the end of the loan?

b. Suppose it were possible to invest in a two-year $10,000 CD at an APR of 8.75% compounded monthly. How much interest would you be paid by the end of the period?

c. In part a, the financial institution loaned you $10,000 for two years, but in part b, you loaned the financial institution $10,000 for two years. What is the difference in the amount of interest paid?

22. Down payment. In this exercise, we examine the benefits of making a down payment.

a. You want to buy a car. Suppose you borrow $15,000 for two years at an APR of 6%. What will your monthly payment be? How much interest will you have paid by the end of the loan?

b. Suppose that in the situation of part a, you make a down payment of $2000. This means that you borrow only $13,000. Assume that the term is still two years at an APR of 6%. What will your monthly payment be? How much interest will you have paid by the end of the loan?

c. Compare your answers to parts a and b. What are the advantages of making a down payment? Why would a borrower not make a down payment? *Note*: One other factor to consider is the interest one would have earned on the down payment if it had been invested. Typically, the interest rate earned on investments is lower than that charged for loans.

23. Amortization table. Suppose we borrow $1500 at 4% APR and pay it off in 24 monthly payments. Make an amortization table showing payments over the first three months.

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				$1500.00
1				
2				
3				

24. Another amortization table. Suppose we borrow $100 at 5% APR and pay it off in 12 monthly payments. Make an amortization table showing payments over the first three months.

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance
				$100.00
1				
2				
3				

25. Term of mortgage. There are two common choices for the term of a home mortgage: 15 or 30 years. Suppose you need to borrow $90,000 at an APR of 6.75% to buy a home.

a. What will your monthly payment be if you opt for a 15-year mortgage?

b. What percentage of your first month's payment will be interest if you opt for a 15-year mortgage? (Round your answer to two decimal places as a percentage.)

c. How much interest will you have paid by the end of the 15-year loan?

d. What will your monthly payment be if you opt for a 30-year mortgage?

e. What percentage of your first month's payment will be interest if you opt for a 30-year mortgage? (Round your answer to two decimal places as a percentage.)

f. How much interest will you have paid by the end of the 30-year loan? Is it twice as much as for a 15-year mortgage?

26. Formula for equity. Here is a formula for the equity built up after k monthly payments:

$$\text{Equity} = \frac{\text{Amount borrowed} \times ((1+r)^k - 1)}{((1+r)^t - 1)}$$

where r is the monthly interest rate as a decimal and t is the term in months. In this exercise, we consider a mortgage of $100,000 at an APR of 7.2% with two different terms.

a. Assume that the term of the mortgage is 30 years. How much equity will you have halfway through the term of the loan? What percentage of the principal is this? (Round your answer to one decimal place as a percentage.)

b. Suppose now that instead of a 30-year mortgage, you have a 15-year mortgage. Find your equity halfway through the term of the loan, and find what percentage of the principal that is. (Round your answer to one decimal place as a percentage.) Compare this with the percentage you found in part a. Why should the percentage for the 15-year term be larger than the percentage for the 30-year term?

27. Car totaled. In order to buy a new car, you finance $20,000 with no down payment for a term of five years at an APR of 6%. After you have the car for one year, you are in an accident. No one is injured, but the car is totaled. The insurance company says that before the accident, the value of the car had decreased by 25% over the time you owned it, and the company pays you that depreciated amount after subtracting your $500 deductible.

a. What is your monthly payment for this loan?

b. How much equity have you built up after one year? *Suggestion*: Use the formula for equity stated in connection with Exercise 26.

c. How much money does the insurance company pay you? (Don't forget to subtract the deductible.)

d. Can you pay off the loan using the insurance payment, or do you still need to make payments on a car you no longer have? If you still need to make payments, how much do you still owe? (Subtract the payment from the insurance company.)

28. Rebates. When interest rates are low, some automobile dealers offer loans at 0% APR, as indicated in a 2021 advertisement by a prominent car dealership, offering zero percent financing or cash back deals on some models.

Zero percent financing means the obvious thing—that no interest is being charged on the loan. So if we borrow $1200 at 0% interest and pay it off over 12 months, our monthly payment will be $\$1200/12 = \$100$.

Suppose you are buying a new truck at a price of $20,000. You plan to finance your purchase with a loan that you will repay over two years. The dealer offers two options: either dealer financing with 0% interest, or a $2000 rebate on the purchase price. If you take the rebate, you will have to go to the local bank for a loan (of $18,000) at an APR of 6.5%.

a. What would your monthly payment be if you used dealer financing?

b. What would your monthly payment be if you took the rebate?

c. Should you take the dealer financing or the rebate? How much would you save over the life of the loan by taking the option you chose?

29. Too good to be true? A friend claims to have found a really great deal at a local loan agency without a street address: The agency claims that its rates are so low you can borrow $10,000 with a term of three years for a monthly payment of $200. Is this too good to be true? Be sure to explain your answer.

30. Is this reasonable? A lending agency advertises in the paper an APR of 12% on a home mortgage with a term of 30 years. The ad claims that the monthly payment on a principal of $100,000 will be $10,290. Is this claim reasonable? What should the ad have said the payment would be (to the nearest dollar)? What do you think happened here?

Exercise 31 is suitable for group work.

31. Affordability. Over the past 40 years, interest rates have varied widely. The rate for a 30-year mortgage reached a high of 14.75% in July 1984, and it reached 3.31%, in November, 2012. A significant impact of lower interest rates on society is that they enable more people to afford the purchase of a home. In this exercise, we consider the purchase of a home that sells for $125,000. Assume that we can make a down payment of $25,000, so we need to borrow $100,000. We assume that our annual income is $40,000 and that we have no other debt. We determine whether we can afford to buy the home at the high and low rates mentioned at the beginning of this paragraph.

a. What is our monthly income?

b. Lending agencies usually require that no more than 28% of the borrower's monthly income be spent on housing. How much does that represent in our case?

c. The amount we will spend on housing consists of our monthly mortgage payment plus property taxes and hazard insurance. Assume that property taxes plus insurance total $250 per month, and subtract this from the answer to part b to determine what monthly payment we can afford.

d. Use your answer to part c to determine how much we can borrow if the term is 30 years and the interest rate is at the historic high of 14.75%. Can we afford the home?

e. Use your answer to part c to determine how much we can borrow if the term is 30 years and the interest rate is 3.31%. Can we afford the home now?

f. What is the difference in the amount we can borrow between the rates used in parts d and e?

32–33. Payday loans. *Exercises 32 and 33 concern the In the News 4.4 article found at paydayloansonlineresource.org. The article explains what are called payday loans and points out their dangers.*

32. Suppose you borrow $100 from a payday lender for one week at a weekly rate of 10%. You'll obviously owe $110 at the end of a week. If you are unable to repay the loan, however, the lender will say that you now owe not only $110 but also 10% of that $110 at the end of the second week. How much will you owe by the end of the second week? If you *still* can't repay any of it, how much will you owe at the end of the third week?

33. Under the same scenario as in Exercise 32, it turns out that after n weeks of not repaying anything you would owe 100×1.1^n dollars. Use this formula to determine how much you would owe after 10 weeks. What total percent interest are you being charged on your 10-week loan? (Round to the nearest percent.)

34. Russian payday loans. An article in *The New York Times* on April 29, 2016, covered payday loans in Russia. According to the article, a private "microfinance industry" extends very small loans to people. What is remarkable is that the lenders sometimes hire thugs to physically assault borrowers if they do not pay. The payday loans in Russia average $125, according to United Credit Bureau statistics, and the interest rate is usually 2% per day.

a. If a person borrowed $100 at 2% per day simple interest, what would she owe after a year?

b. What would she owe if interest were compounded daily?

The following exercises are designed to be solved using technology such as calculators or computer spreadsheets.

35. Equity. You borrow $15,000 with a term of four years at an APR of 8%. Make an amortization table. How much equity have you built up halfway through the term?

36. Finding the term. You want to borrow $15,000 at an APR of 7% to buy a car, and you can afford a monthly payment of $500. To minimize the amount of interest paid, you want to take the shortest term you can. What is the shortest term you can afford? *Note*: Your answer should be a whole number of years. Rule of Thumb 4.2 should give you a rough idea of what the term will be.

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Average Interest Rates for Payday Loans

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If you are strapped for cash and are considering taking out a payday loan, there are several things you should first consider, such as how high the fees and interest rates associated with your loan are. Oftentimes with payday loans, the rates are much higher than other types of loans, and can end up putting you more in debt than you were to start with.

Payday loans typically range from approximately $100 to $1000, depending upon your state's legal minimum. The average loan time is two weeks, after which time you would have to repay the loan along with the fees and interest you accrued over that period. These loans usually cost 400% annual interest (APR), if not more. And the finance charge to borrow $100 ranges from $15 to $30 for two-week loans. These finance charges are sometimes accompanied by interest rates ranging from 300% to 750% APR. For loans shorter than two weeks, the APR can be even higher.

In most cases, payday loans are much more expensive than other cash loans. For example, a $500 cash advance on an average credit card that is repaid in one month would cost you $13.99 in finance charges and an annual interest rate of about 5.7%. A payday loan, on the other hand, would cost you $17.50 per $100 for borrowing the same $500, and would cost $105 if renewed once, or 400% annual interest.

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Writing About Mathematics

37. History of home ownership. In the United States before the 1930s, home ownership was not standard—most people rented. In part, this was because home loans were structured differently: A large down payment was required, the term of the loan was five years or less, the regular payments went toward interest, and the principal was paid off in a lump sum at the end of the term. Home mortgages as we know them came into being through the influence of the Federal Housing Administration, established by Congress in the National Housing Act in 1934, which provided insurance to lending agencies. Find more details on the history of home mortgages, and discuss why the earlier structure of loans would discourage home ownership. Does the earlier structure have advantages?

38. Microloans and flat interest. The 2006 Nobel Peace Prize was awarded to Muhammad Yunus and the Grameen Bank he founded. The announcement of the award noted the development of *microloans* for encouraging the poor to become entrepreneurs. Microloans involve small amounts of money but fairly high interest rates. In contrast to the installment method discussed in this section, for microloans typically interest is paid at a flat rate: The amount of interest paid is not reduced as the principal is paid off. More recently such loans have come under some criticism. Investigate flat-rate loans, the reasons why microloans are often structured this way, and any downsides to them.

39. More on payday loans. Go on the Internet and explore some actual payday loan companies and also what regulations may exist. The Web site http://usgovinfo.about.com/od/consumerawareness/a/paydayloans.htm is a good place to learn more about payday loans.

40. Local payday loans. Explore some actual payday loan companies in your area. Write a report on their rates and policies. Some lenders require the borrower to allow them to withdraw payments from the borrower's bank account. See if companies in your area do this.

41. Local auto loan rates. Find out what some automobile loan rates are in your area. Compare them with some you find online and write a report on your findings.

42. College loan rates. Borrowing for college is very common. Investigate college loan rates and write a report on your findings. If you can find out what rate some of your friends are paying, you might include that.

43. Title loans. Look up the term *title loan*. Write a report on what title loans are and the rates and policies of companies that offer them.

What Do You Think?

44. Is that the right payment? You are negotiating a loan with a bank officer. The loan is at a moderate interest rate, and the term of the loan is six months. The bank officer tells you your monthly payment. Explain how you can quickly check whether or not the figure the bank officer gave you is reasonable. **Exercise 15 is relevant to this topic.**

45. Is this true? You are considering buying a home, but a friend tells you that by the time you finish paying for the home, the amount you pay in interest to the bank will exceed the listed price for the home. Is this an unusual situation? What factors contribute to determining the part of your total payments that is interest? **Exercise 20 is relevant to this topic.**

46. Smart shopping. List the factors that determine the monthly payment for a car loan. Which of these can you control? Experiment by changing each factor to see the effect. For example, if the amount you borrow is doubled, what happens to the monthly payment? Use the information you gather in this way to sketch how you would make a decision about a loan. **Exercise 22 is relevant to this topic.**

47. Amortization table. Explain what an amortization table is. Provide an example. **Exercise 23 is relevant to this topic.**

48. What part of it is mine? You are making monthly installment payments on a car. Explain what is meant by your *equity* in the car. **Exercise 26 is relevant to this topic.**

49. Adjustable-rate mortgage. Discuss the advantages and disadvantages of an adjustable-rate mortgage versus a fixed-rate mortgage.

50. Is that the right payment? You are negotiating a home mortgage that has a term of 30 years. The interest rate is moderate. You are presented with a monthly payment figure that does not include taxes and insurance. Explain how you can quickly check whether the figure the bank officer gave you is reasonable. **Exercise 30 is relevant to this topic.**

51. Research on taxes and insurance. When you take out a mortgage to buy a home, the lending institution will require you to have insurance and to pay your property taxes. The amount of these costs varies depending on your location. Fees to cover these costs are normally added to your monthly payment. These fees are placed in an *escrow account*, which is used to pay the bills when they come due. Figure 4.9 on page 244 shows a mortgage statement with the escrow listed. How much is it? Assume a home is assessed at $100,000. Investigate the annual cost of insurance and property taxes on this home in your area. **Exercise 31 is relevant to this topic.**

52. Research on closing costs. When you secure a home mortgage, the lending institution will charge you *closing costs*. These are fees in addition to the price of the home, interest on the loan, taxes, and insurance. These fees fall into several categories, and they can be substantial. Use the Internet to investigate closing costs. Report your findings.

53. Research on Fannie and Freddie. Fannie Mae and Freddie Mac came under scrutiny during the mortgage crisis of 2007–2008. Use the Internet to determine what these entities are and what role they play in home mortgages. Report your findings.

4.3 Saving for the long term: Build that nest egg

TAKE AWAY FROM THIS SECTION
Prepare for the future by saving now.

The following article was found on the MoneyNing Web site.

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4 Retirement Tips Young Adults Can Implement NOW

JESSICA SOMMERFIELD

Recent studies have shown that a large percentage of young adults under 35 are declining enrollment in employer-provided retirement plans. Considering the economy, it's understandable that many young adults are finding it impossible to focus on things any further off than paying student loan debt, purchasing a home or vehicle, and getting started in their careers.

Many won't be financially able to leave their parents' home until well into their twenties. **Even if they're interested, today's young adults find it hard to sacrifice a portion of their often meager wages for retirement savings.** Especially if they're barely getting by as it is.

Considering this trend in the next generation, some companies that employ mostly younger adults aren't bothering to offer retirement plan options and are choosing instead to pay higher wages. While this may be saving companies money and allowing Generation Y or Millennial adults to channel their money into immediate debt-payoff or other savings goals, it's not encouraging the younger generations to think about and plan for the future.

Market research shows that those who are actively saving and investing for their retirement are in the 35–65 age bracket. But, according to many financial experts, this is alarmingly late in the game. **The obvious dilemma is that young adults need to start saving for retirement as soon as possible—while not floundering under other financial obligations.** It's a challenge, but it can be done.

Here are four retirement tips that young adults can start implementing now.

1. Invest early, no matter how small

Time is on your side. Because of the wonder of compound interest, as little as $200 a month invested in your twenties can build to millions by the time you retire. But realistically, how many young adults in our economy have $200 a month to put into retirement savings?

The recommended allocation for retirement savings is 15% of your income, but this is often an unrealistic number. **The key is to stop worrying about percentages and focus on saving *something*.** Even small contributions to a 401K or Roth IRA when you're young will put you much further ahead. You can always invest more heavily once you're in a better financial situation.

2. If your employer doesn't offer a 401K, open an IRA

If your employer offers a 401K, you should undoubtedly participate. Most employers match your contribution, sometimes up to 100%. **Investments are before-tax and non-taxable, unless you withdraw them early.**

If they don't offer a 401K, consider opening a Roth IRA. Although investment companies usually require a minimum amount to start an account, many are catching on to the needs of young adults and waving the minimum as long as you agree to regular contributions. Again, the amount isn't as important as the fact that you're saving something.

(*Continued*)

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4 Retirement Tips (*Continued*)

3. Keep your 401K untouched

When you're strapped for money, it can be tempting to consider your 401K funds as just another savings account to withdraw from in an emergency. But if you do so, you'll be paying a 10% early withdrawal fee, plus a heavy fee of up to 20% to your employer for income tax. **The amount you'll receive is not worth the amount you'll lose.**

If you change jobs, your new employer might allow you to roll your previous 401K over into theirs. If not, you should open a Roth IRA and transfer the funds to that.

4. Remember that the future is important

Don't get so caught up in the present that you neglect to plan for the future. This is one of the greatest failings of the two youngest generations. The transition into adulthood and financial independence is becoming more difficult in the present circumstances—but **neglecting to plan for the future can result in a lack of financial independence later in life**. Don't be discouraged if you can't save much right now; saving something is always better than nothing.

Young adults, how have you begun saving for retirement? Any tips?

In Section 4.1, we discussed how an account grows if a lump sum is invested. Many long-term savings plans such as retirement accounts combine the growth power of compound interest with that of regular contributions. Such plans can show truly remarkable growth. We look at such accounts in this section.

". . . and help my parents to pick the right investments for my college education."

Saving regular amounts monthly

Let's look at a plan where you deposit \$100 to your savings account at the end of each month, and let's suppose, for simplicity, that the account pays you a monthly rate of 1% on the balance in the account. (That is an APR of 12% compounded monthly.)

At the end of the first month, the balance is \$100. At the end of the second month, the account is increased by two factors—interest earned and a second deposit. Interest earned is 1% of \$100 or \$1.00, and the deposit is \$100. This gives the new balance as

$$\begin{aligned}\text{New balance} &= \text{Previous balance} + \text{Interest} + \text{Deposit}\\ &= \$100 + \$1 + \$100 = \$201\end{aligned}$$

Table 4.2 tracks the growth of this account through 10 months. Note that at the end of each month, the interest is calculated on the previous balance and then \$100 is added to the balance.

TABLE 4.2 Regular Deposits into an Account

At end of month number	Interest paid on previous balance	Deposit	Balance
1	\$0.00	\$100	\$100.00
2	1% of \$100.00 = \$1.00	\$100	\$201.00
3	1% of \$201.00 = \$2.01	\$100	\$303.01
4	1% of \$303.01 = \$3.03	\$100	\$406.04
5	1% of \$406.04 = \$4.06	\$100	\$510.10
6	1% of \$510.10 = \$5.10	\$100	\$615.20
7	1% of \$615.20 = \$6.15	\$100	\$721.35
8	1% of \$721.35 = \$7.21	\$100	\$828.56
9	1% of \$828.56 = \$8.29	\$100	\$936.85
10	1% of \$936.85 = \$9.37	\$100	\$1046.22

EXAMPLE 4.20 Verifying a balance: Regular deposits into savings

Verify the calculation shown for month 3 of Table 4.2.

SOLUTION

We earn 1% interest on the previous balance of \$201.00:

$$\text{Interest} = 0.01 \times \$201 = \$2.01$$

We add this amount plus a deposit of \$100 to the previous balance:

$$\begin{aligned}\text{Balance at end of month 3} &= \text{Previous balance} + \text{Interest} + \text{Deposit}\\ &= \$201.00 + \$2.01 + \$100\\ &= \$303.01\end{aligned}$$

This agrees with the entry in Table 4.2.

TRY IT YOURSELF 4.20

Verify the calculation shown for month 4 of Table 4.2.

The answer is provided at the end of this section.

In Table 4.2, the balance after 10 months is \$1046.22. There were 10 deposits made, for a total of \$1000. The remaining \$46.22 comes from interest.

Suppose we had deposited that entire \$1000 at the beginning of the 10-month period. We use the compound interest formula (Formula 4.3 from Section 4.1) to find the balance at the end of the period. The monthly rate of 1% in decimal form is $r = 0.01$, so we find that our balance would have been

$$\begin{aligned}\text{Balance after 10 months} &= \text{Principal} \times (1+r)^t \\ &= \$1000 \times (1+0.01)^{10} \\ &= \$1104.62\end{aligned}$$

That number is larger than the balance for monthly deposits because it includes interest earned on the full \$1000 over the entire 10 months. Of course, for most of us it's easier to make monthly deposits of \$100 than it is to come up with a lump sum of \$1000 to invest.

These observations provide a helpful rule of thumb even though we don't yet have a formula for the balance.

RULE OF THUMB 4.4 Regular Deposits

Suppose we deposit money regularly into an account with a fixed interest rate.

1. The ending balance is at least as large as the total deposit.
2. The ending balance is less than the amount we would have if the entire deposit were invested initially and allowed to draw interest over the whole term.
3. These estimates are best over a short term.

Regular deposits balance formula

The formula for the account balance, assuming regular deposits at the end of each month, is as follows.[9]

FORMULA 4.8 Regular Deposits Balance Formula

$$\text{Balance after } t \text{ monthly deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

where t is the number of deposits, and r is the monthly interest rate APR/12 expressed as a decimal.

Alternatively, we can write the formula as

$$\text{Balance after deposits for } y \text{ years} = \frac{\text{Deposit} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12}\right)}$$

where deposits are made monthly and y is the number of years. This form is equivalent since $r = \frac{\text{APR}}{12}$ and the number of deposits is $t = 12 \times y$.

[9] *See Exercise 23 for the adjustment necessary when deposits are made at the beginning of the month.*

The ending balance is often called the *future value* for this savings arrangement. A derivation of this formula is given in Algebraic Spotlight 4.4 at the end of this section. Throughout this section, we assume that interest is compounded monthly.

Let's check that this formula agrees with the balance at the end of the tenth month as found in Table 4.2. The deposit is \$100 each month, and the monthly rate as a decimal is $r = 0.01$. We want the balance after $t = 10$ deposits. Using Formula 4.8, we find

$$\begin{aligned}\text{Balance after 10 deposits} &= \frac{\text{Deposit} \times ((1+r)^t - 1)}{r} \\ &= \frac{\$100 \times (1.01^{10} - 1)}{0.01} \\ &= \$1046.22\end{aligned}$$

This is the same as the answer we obtained in Table 4.2.

EXAMPLE 4.21 Using the balance formula: Saving money regularly

Suppose we have a savings account earning 7% APR. We deposit \$20 to the account at the end of each month for five years. What is the future value for this savings arrangement? That is, what is the account balance after five years?

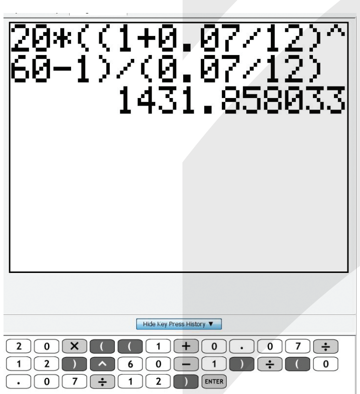
SOLUTION

We use the regular deposits balance formula (Formula 4.8). The monthly deposit is \$20, and the monthly interest rate as a decimal is

$$r = \frac{\text{APR}}{12} = \frac{0.07}{12}$$

The number of deposits is $t = 5 \times 12 = 60$. The regular deposits balance formula gives

$$\begin{aligned}\text{Balance after 60 deposits} &= \frac{\text{Deposit} \times ((1+r)^t - 1)}{r} \\ &= \frac{\$20 \times ((1+0.07/12)^{60} - 1)}{(0.07/12)} \\ &= \$1431.86\end{aligned}$$



Benny Evans

The future value is \$1431.86.

TRY IT YOURSELF 4.21

Suppose we have a savings account earning 4% APR. We deposit \$40 to the account at the end of each month for 10 years. What is the future value of this savings arrangement?

The answer is provided at the end of this section.

Determining the savings needed

People approach savings in different ways: Some are committed to depositing a certain amount of money each month into a savings plan, and others save with a specific purchase in mind. Suppose you are nearing the end of your sophomore year and plan to purchase a car when you graduate in two years. If the car you have your eye on will cost \$20,000 when you graduate, you want to know how much you will have to save each month for the next two years to have \$20,000 at the end.

We can rearrange the regular deposits balance formula to tell us how much we need to deposit regularly in order to achieve a goal (that is, a future value) such as this.

FORMULA 4.9 Deposit Needed Formula

$$\text{Needed monthly deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

where r is the monthly interest rate APR/12 as a decimal, and t is the number of deposits you will make to reach your goal.

Alternatively, we can write the formula as

$$\text{Needed monthly deposit} = \frac{\text{Goal} \times \left(\frac{\text{APR}}{12}\right)}{\left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

where y is the number of years to reach your goal. This form is equivalent since $r = \frac{\text{APR}}{12}$ and the number of deposits is $t = 12 \times y$.

For example, if your goal is \$20,000 to buy a car in two years, and if the APR is 6%, you can use this formula to find how much you need to deposit each month. The monthly rate as a decimal is

$$r = \frac{\text{APR}}{12} = \frac{0.06}{12} = 0.005$$

We use $t = 2 \times 12 = 24$ deposits in the formula:

$$\begin{aligned} \text{Needed deposit} &= \frac{\text{Goal} \times r}{((1+r)^t - 1)} \\ &= \frac{\$20{,}000 \times 0.005}{(1.005^{24} - 1)} \\ &= \$786.41 \end{aligned}$$

You need to deposit \$786.41 each month so you can buy that \$20,000 car when you graduate.

Student debt is today a significant issue for students and in the United States as a whole. The following excerpt of an article from the Web site Demos shows the importance of saving for college.

IN THE NEWS 4.6

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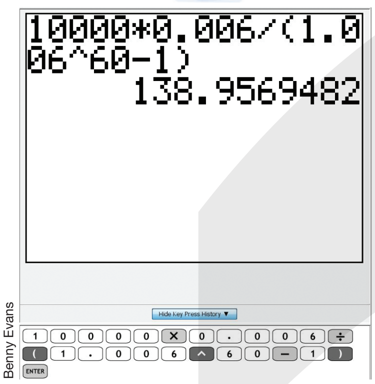
AT WHAT COST? How Student Debt Reduces Lifetime Wealth

By ROBERT HILTONSMITH

Student debt has skyrocketed over the past decade, quadrupling from just \$240 billion in 2003 to more than \$1 trillion today [2013]. If current borrowing patterns continue, student debt levels will reach \$2 trillion in 2025. Average debt levels have risen rapidly as well: two-thirds (66 percent) of college seniors now graduate with an average of \$26,600 in student loans, up from 41 percent in 1989. The rise of this “debt-for-diploma” system over the past decade was largely caused by the sharp decline in state funding for higher education, which has fallen by 25 percent since its peak in 2000.

Hiltonsmith, Robert, “At what cost?: How student debt reduces lifetime wealth,” Demos



Benny Evans

EXAMPLE 4.22 Computing deposit needed: Saving for college

How much does your younger brother need to deposit each month into a savings account that pays 7.2% APR in order to have \$10,000 when he starts college in five years?

SOLUTION

Your brother wants to achieve a goal of \$10,000 in five years, so he uses Formula 4.9. The monthly interest rate as a decimal is

$$r = \frac{\text{APR}}{12} = \frac{0.072}{12} = 0.006$$

and the number of deposits is $t = 5 \times 12 = 60$:

$$\begin{aligned}\text{Needed deposit} &= \frac{\text{Goal} \times r}{((1+r)^t - 1)}\\ &= \frac{\$10{,}000 \times 0.006}{(1.006^{60} - 1)}\\ &= \$138.96\end{aligned}$$

He needs to deposit \$138.96 each month.

TRY IT YOURSELF 4.22

How much do you need to deposit each month into a savings account that pays 9% APR in order to have \$50,000 for your child to use for college in 18 years?

The answer is provided at the end of this section.

SUMMARY 4.5 Monthly Deposits

Suppose we deposit a certain amount of money at the end of each month into a savings account that pays a monthly interest rate of $r = \text{APR}/12$ as a decimal. The balance in the account after t months is given by the regular deposits balance formula (Formula 4.8):

$$\text{Balance after } t \text{ monthly deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

The ending balance is called the future value for this savings arrangement.

A companion formula (Formula 4.9) gives the monthly deposit necessary to achieve a given balance:

$$\text{Needed monthly deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

Alternatively, we can write these formulas in term of y, the number of years:

$$\text{Balance after monthly deposits for } y \text{ years} = \frac{\text{Deposit} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12}\right)}$$

and

$$\text{Needed monthly deposit} = \frac{\text{Goal} \times \left(\frac{\text{APR}}{12}\right)}{\left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

Saving for retirement

As the article at the beginning of this section points out, college students often don't think much about retirement, but early retirement planning is important.

EXAMPLE 4.23 Finding deposit needed: Retirement and varying rates

Suppose that you'd like to retire in 40 years and want to have a future value of \$500,000 in a savings account. (See the article at the beginning of this section.) Also suppose that your employer makes regular monthly deposits into your retirement account.

a. If you can expect an APR of 9% for your account, how much do you need your employer to deposit each month?

b. The formulas we have been using assume that the interest rate is constant over the period in question. Over a period of 40 years, though, interest rates can vary widely. To see what difference the interest rate can make, let's assume a constant APR of 6% for your retirement account. How much do you need your employer to deposit each month under this assumption?

SOLUTION

a. We have a goal of \$500,000, so we use Formula 4.9. The monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.09}{12} = 0.0075$$

The number of deposits is $t = 40 \times 12 = 480$, so the needed deposit is

$$\begin{aligned}\text{Needed deposit} &= \frac{\text{Goal} \times r}{((1+r)^t - 1)} \\ &= \frac{\$500{,}000 \times 0.0075}{(1.0075^{480} - 1)} \\ &= \$106.81\end{aligned}$$

b. The computation is the same as in part a except that the new monthly rate as a decimal is

$$r = \frac{0.06}{12} = 0.005$$

We have

$$\begin{aligned}\text{Needed deposit} &= \frac{\text{Goal} \times r}{((1+r)^t - 1)} \\ &= \frac{\$500{,}000 \times 0.005}{(1.005^{480} - 1)} \\ &= \$251.07\end{aligned}$$

Note that the decrease in the interest rate from 9% to 6% requires that you more than double your monthly deposit if you are to reach the same goal. The effect of this possible variation in interest rates is one factor that makes financial planning for retirement complicated.

Retirement income: Annuities

How much income will you need in retirement? That's a personal matter, but we can analyze what a *nest egg* will provide.

KEY CONCEPT

Your nest egg is the balance of your retirement account at the time of retirement. The monthly yield is the amount you can withdraw from your retirement account each month.

Once you retire, there are several ways of using your retirement funds. One method is to withdraw each month only the interest accrued over that month; the principal remains the same. Under this arrangement, your nest egg will never be reduced; you'll be living off the interest alone. An arrangement like this is called a *perpetuity* because the constant income continues indefinitely. You can explore this arrangement further in Exercises 24 through 30.

With a perpetuity, the original balance at retirement remains untouched, but if we are willing to reduce the principal each month, we won't need to start with as large a nest egg for our given monthly income. In this situation, we receive a constant monthly payment, part of which represents interest and part of which represents a reduction of principal. Such an arrangement is called a *fixed-term annuity* because, unlike a perpetuity, this arrangement will necessarily end after a fixed term (when we have spent the entire principal).

KEY CONCEPT

An annuity is an arrangement that withdraws both principal and interest from your nest egg. Payments end when the principal is exhausted.

An annuity works just like the installment loans we considered in Section 4.2, only someone is paying *us* rather than the other way around.[10] In fact, the formula for the monthly payment applies in this situation, too. We can think of the institution that holds our account at retirement as having borrowed our nest egg; it will pay us back in monthly installments over the term of the annuity.

FORMULA 4.10 Annuity Yield Formula

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times r \times (1+r)^t}{((1+r)^t - 1)}$$

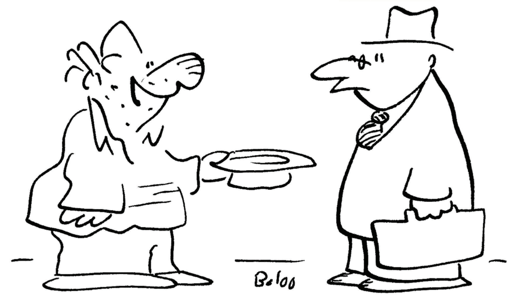
where r is the monthly rate (as a decimal), and t is the term (the number of months the annuity will last).

Alternatively, we can write the formula as

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times \dfrac{\text{APR}}{12} \times \left(1 + \dfrac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1 + \dfrac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

where y is the term of the annuity in years.

[10] *We assume that the payments are made at the end of the month.*


cartoonstock.com

"We could both avoid this daily annoyance, sir, if you'd buy me an *annuity!*"

EXAMPLE 4.24 Finding annuity yield: 20-year annuity

Suppose we have a nest egg of \$800,000 with an APR of 6% compounded monthly. Find the monthly yield for a 20-year annuity.

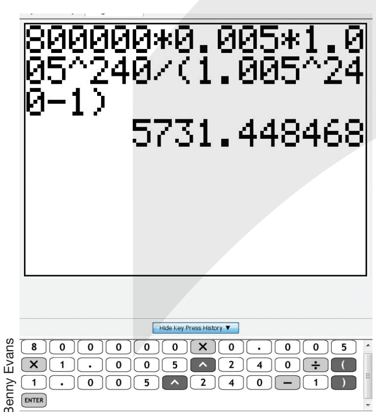
SOLUTION

We use the annuity yield formula (Formula 4.10). With an APR of 6%, the monthly rate as a decimal is

$$r = \text{Monthly rate} = \frac{\text{APR}}{12} = \frac{0.06}{12} = 0.005$$

The term is 20 years, so we take $t = 20 \times 12 = 240$ months:

$$\begin{aligned} \text{Monthly annuity yield} &= \frac{\text{Nest egg} \times r \times (1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$800{,}000 \times 0.005 \times 1.005^{240}}{(1.005^{240} - 1)} \\ &= \$5731.45 \end{aligned}$$



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TRY IT YOURSELF 4.24

Suppose we have a nest egg of \$1,000,000 with an APR of 6% compounded monthly. Find the monthly yield for a 25-year annuity.

The answer is provided at the end of this section.

How large a nest egg is needed to achieve a desired annuity yield? We can answer this question by rearranging the annuity yield formula (Formula 4.10).

FORMULA 4.11 Annuity Yield Goal

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

where r is the monthly interest rate and t is the term of the annuity in months.

Alternatively, we can write the formula as

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

where y is the term of the annuity in years.

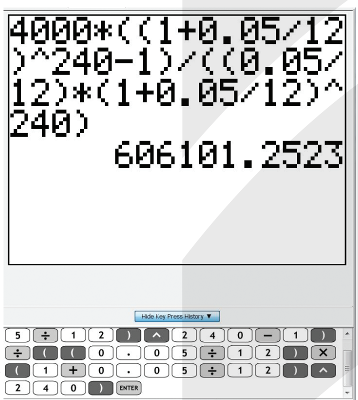
Here, r is the monthly rate (as a decimal), and t is the term (in months) of the annuity.

The following example shows how we use this formula.

EXAMPLE 4.25 Finding nest egg needed for annuity: Retiring on a 20-year annuity

Suppose our retirement account pays 5% APR compounded monthly. What size nest egg do we need in order to retire with a 20-year annuity that yields \$4000 per month?

SOLUTION



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We want to achieve an annuity goal, so we use Formula 4.11. The monthly rate as a decimal is $r = 0.05/12$, and the term is $t = 20 \times 12 = 240$ months:

$$\begin{aligned}\text{Nest egg needed} &= \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)} \\ &= \frac{\$4000 \times ((1+0.05/12)^{240} - 1)}{((0.05/12) \times (1+0.05/12)^{240})} \\ &= \$606{,}101.25\end{aligned}$$

TRY IT YOURSELF 4.25

Suppose our retirement account pays 9% APR compounded monthly. What size nest egg do we need in order to retire with a 25-year annuity that yields \$5000 per month?

The answer is provided at the end of this section.

The balance at retirement (the nest egg) is called the *present value* of the annuity. Future value and present value depend on perspective. When you started saving, the balance at retirement was the future value. When you actually retire, it becomes the present value.

The obvious question is, how many years should you plan for the annuity to last? If you set it up to last until you're 80 and then you live until you're 85, you're in trouble. What a retiree often wants is to have the monthly annuity payment continue for as long as he or she lives. Insurance companies offer such an arrangement, and it's called a *life annuity*. How does it work?

The insurance company makes a statistical estimate of the life expectancy of a customer, which is used in determining how much the company will probably have to pay out.[11] Some customers will live longer than the estimate (and the company

[11] *Such calculations are made by professionals known as* actuaries.

may lose money on them) but some will not (and the company will make money on them). The monthly income for a given principal is determined from this estimate using the formula for the present value of a fixed-term annuity.

SUMMARY 4.6
Retiring on an Annuity

For a fixed-term annuity, the formulas for monthly payments apply. Let r be the monthly interest rate (as a decimal) and t the term (in months) of the annuity.

1. To find the monthly yield provided by a nest egg, we use

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times r \times (1+r)^t}{((1+r)^t - 1)}$$

2. To find the nest egg needed to provide a desired income, we use

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

The balance at retirement (the nest egg) is called the present value of the annuity.

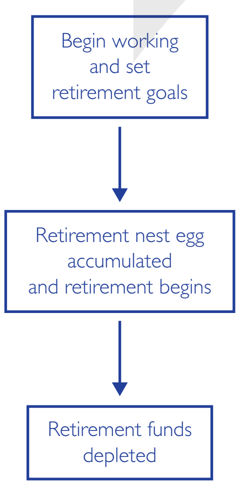
Alternatively, we can write these formulas in terms of y, the term of the annuity in years:

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times \frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}$$

and

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times \left(\left(1 + \frac{\text{APR}}{12}\right)^{(12y)} - 1\right)}{\left(\frac{\text{APR}}{12} \times \left(1 + \frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

The following schematic illustrates how the formulas in this section apply to your long-term financial security. Financial advisors emphasize the importance of making an early beginning in saving for retirement.



Steps to be taken at the beginning of your working life to prepare for retirement

- Set goal for monthly income at retirement.
- Use the annuity yield goal formula (Formula 4.11) to determine the nest egg needed to meet your income goal. (Take into account income from other sources, such as Social Security.)
- Use the deposit needed formula (Formula 4.9) to determine the monthly savings needed to meet your goal.

Steps to be taken at the time of retirement

- Assess the actual accumulated nest egg.
- Use the annuity yield formula (Formula 4.10) to determine your actual monthly income at retirement.

ALGEBRAIC SPOTLIGHT 4.4 Derivation of the Regular Deposits Balance Formula

Suppose that at the end of each month, we deposit money (the same amount each month) into an account that pays a monthly rate of r as a decimal. Our goal is to derive the formula

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

In Algebraic Spotlight 4.2 from Section 4.2, we considered the situation where we borrow B_0 dollars and make monthly payments of M dollars at a monthly interest rate of r as a decimal. We found that the balance after t payments is

$$\text{Balance after } t \text{ payments} = B_0(1+r)^t - M\frac{(1+r)^t - 1}{r}$$

Making payments of M dollars per month subtracts money from the balance. A deposit adds money rather than subtracting it. The result is to add the term involving M instead of subtracting it:

$$\text{Balance after } t \text{ deposits} = B_0(1+r)^t + M\frac{(1+r)^t - 1}{r}$$

Now the initial balance is zero, so $B_0 = 0$. Therefore,

$$\text{Balance after } t \text{ deposits} = M\frac{(1+r)^t - 1}{r}$$

Because M is the amount we deposit, we can write the result as

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

WHAT DO YOU THINK?

Your daughter's college education: You figure you will need \$50,000 to pay for your baby daughter's college education. You want to start saving now. Do you need to immediately invest \$50,000? Explain how you might be able to invest less.

Questions of this sort may invite many correct responses. We present one possibility: You can invest a good deal less than \$50,000 because your investment will be earning interest. If interest is being compounded, and if you start while your daughter is young, you may be able to invest significantly less than \$50,000. This is because the compound interest contributes to exponential growth, and as we know, exponential growth can be dramatic over the long term.

Further questions of this type may be found in the *What Do You Think?* section of exercises.

Try It Yourself answers

Try It Yourself 4.20: Verifying a balance: Regular deposits into an account
$\$303.01 + 0.01 \times \$303.01 + \$100 = \$406.04$.

Try It Yourself 4.21: Using the balance formula: Saving money regularly \$5889.99.

Try It Yourself 4.22: Computing deposit needed: Saving for college \$93.22.

Try It Yourself 4.24: Finding annuity yield: 25-year annuity \$6443.01.

Try It Yourself 4.25: Finding nest egg needed for annuity: Retiring on a 25-year annuity \$595,808.11.

Exercise Set 4.3

Test Your Understanding

1. Explain how an annuity works.

2. A person's *nest egg* is: **a.** the total amount of money she has at the present **b.** the amount of money she has in savings at age 65 **c.** the amount of money she has in her retirement account at the time she retires **d.** the amount of money she has in her checking account at the time she retires.

3. True or false: In most of the formulas in this section you see $\frac{\text{APR}}{12}$. This is because the formulas are based on monthly interest rates.

4. True or false: Although compounding makes an account grow faster, it is not an important factor in saving for retirement.

5. Explain in your own words what the regular deposits balance formula (Formula 4.8) tells you.

6. Explain in your own words what the deposit needed formula (Formula 4.9) tells you.

Problems

7. Saving for a car. You are saving to buy a car, and you deposit \$200 at the end of each month for two years at an APR of 4.8% compounded monthly. What is the future value for this savings arrangement? That is, how much money will you have for the purchase of the car after two years?

8. Saving for a down payment. You want to save \$20,000 for a down payment on a home by making regular monthly deposits over five years. Take the APR to be 6%. How much money do you need to deposit each month?

9. Planning for college. At your child's birth, you begin contributing monthly to a college fund. The fund pays an APR of 4.8% compounded monthly. You figure that your child will need \$40,000 at age 18 to begin college. What monthly deposit is required?

10. Savings account tabulation. You have a savings account into which you invest \$50 at the end of every month, and the account pays you an APR of 9% compounded monthly.

a. Fill in the following table. (Don't use the regular deposits balance formula.)

At end of month number	Interest paid on prior balance	Deposit	Balance
1			
2			
3			
4			

b. Use the regular deposits balance formula (Formula 4.8) to determine the balance in the account at the end of four months. Compare this to the final balance in the table from part a.

c. Use the regular deposits balance formula to determine the balance in the account at the end of four years.

d. Use the regular deposits balance formula to determine the balance in the account at the end of 20 years.

11. Retirement account tabulation. You have a retirement account into which your employer invests \$75 at the end of every month, and the account pays an APR of 5.25% compounded monthly.

a. Fill in the following table. (Don't use the regular deposits balance formula.)

At end of month number	Interest paid on prior balance	Deposit	Balance
1			
2			
3			
4			

b. Use the regular deposits balance formula (Formula 4.8) to determine the balance in the account at the end of four months. Compare this to the final balance in the table from part a.

c. Use the regular deposits balance formula to determine the balance in the account at the end of four years.

d. Use the regular deposits balance formula to determine the balance in the account at the end of 20 years.

12. Savings account estimations. Jerry calculates that if he makes a deposit of \$5 each month at an APR of 4.8%, then at the end of two years he'll have \$100. Benny says that the correct amount is \$135. Rule of Thumb 4.4 should be helpful in this exercise.

a. What was the total amount deposited (ignoring interest earned)? Whose answer is ruled out by this calculation? Why?

b. Suppose the total amount deposited (\$5 per month for two years) is instead put as a lump sum at the beginning of the two years as principal in an account earning an APR of 4.8%. Use the compound interest formula (with monthly compounding) from Section 4.1 to determine how much would be in the account after two years. Whose answer is ruled out by this calculation? Why?

c. Find the correct balance after two years.

13–14. Saving for a boat. *Suppose you want to save in order to purchase a new boat. Take the APR to be 7.2% in Exercises 13 and 14.*

13. If you deposit \$250 each month, how much will you have toward the purchase of a boat after three years?

14. You want to have \$13,000 toward the purchase of a boat in three years. How much do you need to deposit each month?

15. Fixed-term annuity. You have a 20-year annuity with a present value (that is, nest egg) of \$425,000. If the APR is 7%, what is the monthly yield?

16. Life annuity. You have set up a life annuity with a present value of \$350,000. If your life expectancy at retirement is 21 years, what will your monthly income be? Take the APR to be 6%.

17. Nest egg. You begin working at age 25, and your employer deposits \$300 per month into a retirement account that pays an APR of 6% compounded monthly. You expect to retire at age 65.

a. What will be the size of your nest egg at age 65?

b. Suppose you are allowed to contribute \$100 each month in addition to your employer's contribution. What will be the size of your nest egg at age 65? Compare this with your answer to part a.

18. A different nest egg. You begin working at age 25, and your employer deposits \$250 each month into a retirement account that pays an APR of 6% compounded monthly. You expect to retire at age 65.

a. What will be the size of your nest egg when you retire?

b. Suppose instead that you arranged to start the regular deposits two years earlier, at age 23. What will be the size of your nest egg when you retire? Compare this with your answer from part a.

19. Planning to retire on an annuity. You plan to work for 40 years and then retire using a 25-year annuity. You want to arrange a retirement income of \$4500 per month. You have access to an account that pays an APR of 7.2% compounded monthly.

a. What size nest egg do you need to achieve the desired monthly yield?

b. What monthly deposits are required to achieve the desired monthly yield at retirement?

20. Retiring. You want to have a monthly income of \$2000 from a fixed-term annuity when you retire. Take the term of the annuity to be 20 years and assume an APR of 6% over the period of investment.

a. How large will your nest egg have to be at retirement to guarantee the income described above?

b. You plan to make regular deposits for 40 years to build up your savings to the level you determined in part a. How large must your monthly deposit be?

21. Planning for retirement. You anticipate an average income of \$10,000 per month over your working life. With this in mind, you begin planning for retirement. For this exercise, assume an APR of 5% and round all answers to the nearest whole dollar.

a. Taking into account income from Social Security, you think you will need a monthly income of about 75% of your working income. That is, you want to provide a monthly retirement income of \$7500 for 20 years. Use the annuity yield goal formula (Formula 4.11) to determine the size of the nest egg needed for retirement.

b. Use the deposit needed formula (Formula 4.9) to determine the monthly deposit required to meet the goal from part a. Assume a working life of 45 years.

22. You want to be rich. You plan to work for 50 years before retiring with ten million dollars to spend. Assuming an APR of 6%, how much money do you need to save each month? Round your answer to the nearest whole dollar.

23. Deposits at the beginning. In Section 4.3, we considered the case of regular deposits at the end of each month. If deposits are made at the beginning of each month, then the formula is a bit different. The adjusted formula is

$$\text{Balance after } t \text{ deposits} = \text{Deposit} \times (1+r) \times \frac{((1+r)^t - 1)}{r}$$

Here, r is the monthly interest rate (as a decimal), and t is the number of deposits. The extra factor of $1 + r$ accounts for the interest earned on the deposit over the first month after it's made.

Suppose you deposit \$200 at the beginning of each month for five years. Take the APR to be 7.2%.

a. What is the future value? In other words, what will your account balance be at the end of the period?

b. What would be the future value if you had made the deposits at the end of each month rather than at the beginning? Explain why it is reasonable that your answer here is smaller than that from part a.

24. Retirement income: perpetuities. If a retirement fund is set up as a *perpetuity*, one withdraws each month only the interest accrued over that month; the principal remains the same. For example, suppose you have accumulated \$500,000 in an account with a monthly interest rate of 0.5%. Each month, you can withdraw $\$500{,}000 \times 0.005 = \$2500$ in interest, and the nest egg will always remain at \$500,000. That is, the \$500,000 perpetuity has a monthly yield of \$2500. In general, the monthly yield for a perpetuity is given by the formula

$$\text{Monthly perpetuity yield} = \text{Nest egg} \times \text{Monthly interest rate}$$

In this formula, the monthly interest rate is expressed as a decimal.

Suppose we have a perpetuity paying an APR of 6% compounded monthly. If the value of our nest egg (that is, the present value) is \$800,000, find the amount we can withdraw each month. *Note*: First find the monthly interest rate.

25. Perpetuity yield. *Refer to Exercise 24 for background on perpetuities.* You have a perpetuity with a present value (that is, nest egg) of \$650,000. If the APR is 5% compounded monthly, what is your monthly income?

26. Another perpetuity. *Refer to Exercise 24 for background on perpetuities.* You have a perpetuity with a present value of \$900,000. If the APR is 4% compounded monthly, what is your monthly income?

27. Comparing annuities and perpetuities. *Refer to Exercise 24 for background on perpetuities.* For 40 years, you invest \$200 per month at an APR of 4.8% compounded monthly, then you retire and plan to live on your retirement nest egg.

a. How much is in your account on retirement?

b. Suppose you set up your account as a perpetuity on retirement. What will your monthly income be? (Assume that the APR remains at 4.8% compounded monthly.)

c. Suppose now you use the balance in your account for a life annuity instead of a perpetuity. If your life expectancy is 21 years, what will your monthly income be? (Again, assume that the APR remains at 4.8% compounded monthly.)

d. Compare the total amount you invested with your total return from part c. Assume that you live 21 years after retirement.

28. Perpetuity goal: How much do I need to retire? *Refer to Exercise 24 for background on perpetuities.* Here is the formula for the nest egg needed for a desired monthly yield on a perpetuity:

$$\text{Nest egg needed} = \frac{\text{Desired monthly yield}}{\text{Monthly interest rate}}$$

In this formula, the monthly interest rate is expressed as a decimal.

If your retirement account pays 5% APR with monthly compounding, what present value (that is, nest egg) is required for you to retire on a perpetuity that pays \$4000 per month?

29. Desired perpetuity. *Refer to the formula in Exercise 28.* You want a perpetuity with a monthly income of \$3000. If the APR is 7%, what does the present value have to be?

30. Planning to retire on a perpetuity. You plan to work for 40 years and then retire using a perpetuity. You want to arrange to have a retirement income of \$4500 per month. You have access to an account that pays an APR of 7.2% compounded monthly.

a. *Refer to the formula in Exercise 28.* What size nest egg do you need to achieve the desired monthly yield?

b. What monthly deposits are required to achieve the desired monthly yield at retirement?

Exercise 31 is suitable for group work.

31. Starting early, starting late. In this exercise, we consider the effects of starting early or late to save for retirement. Assume that each account considered has an APR of 6% compounded monthly.

a. At age 20, you realize that even a modest start on saving for retirement is important. You begin depositing \$50 each month into an account. What will be the value of your nest egg when you retire at age 65?

b. Against expert advice, you begin your retirement program at age 40. You plan to retire at age 65. What monthly contributions do you need to make to match the nest egg from part a?

c. Compare your answer to part b with the monthly deposit of \$50 from part a. Also compare the total amount deposited in each case.

d. Let's return to the situation in part a: At age 20, you begin depositing \$50 each month into an account. Now suppose that at age 40, you finally get a job where your employer puts \$400 per month into an account. You continue your \$50 deposits, so from age 40 on, you have two separate accounts working for you. What will be the total value of your nest egg when you retire at age 65?

32. Retiring without interest. Suppose we lived in a society without interest. At age 25, you begin putting \$250 per month into a cookie jar until you retire at age 65. At age 65, you begin to withdraw \$2500 per month from the cookie jar. How long will your retirement fund last?

Exercises 33 and 34 are designed to be solved using technology such as calculators or computer spreadsheets.

33. How much? You begin saving for retirement at age 25, and you plan to retire at age 65. You want to deposit a certain amount each month into an account that pays an APR of 6% compounded monthly. Make a table that shows the amount you must deposit each month in terms of the nest egg you desire to have when you retire. Include nest egg sizes from \$100,000 to \$1,000,000 in increments of \$100,000.

34. How long? You begin working at age 25, and your employer deposits \$350 each month into a retirement account that pays an APR of 6% compounded monthly. Make a table that shows the size of your nest egg in terms of the age at which you retire. Include retirement ages from 60 to 70.

Writing About Mathematics

35. History of annuities. The origins of annuities can be traced back to the ancient Romans. Look up the history of annuities and write a report on it. Include the use of annuities in Rome, in Europe during the seventeenth century, in colonial America, and in modern American society.

36. History of actuaries. Look up the history of the actuarial profession and write a report on it. Be sure to discuss what mathematics are required to become an actuary.

37. Retirement plans. Some people you know, such as family members or teachers, probably have retirement plans. Find out about their plans and write a report on what you learn.

38. Social Security. Most people count on Social Security to provide at least part of their retirement income. Write a report on the history of the Social Security system and how it works.

39. Making a plan. Use what you have learned in this section to create a hypothetical retirement plan for yourself.

What Do You Think?

40. Low interest rate. You are setting up a regular monthly deposit plan but are concerned about the low interest rate you will earn. Should you be more concerned if the plan is for 2 years or if the plan is for 20 years? Explain your reasoning. **Exercise 7 is relevant to this topic.**

41. The regular deposits formula. The regular deposits balance formula is a bit complicated. If you deposit \$100 each month for 10 months, you have deposited a total of \$1000. Use the regular deposits balance formula to calculate the balance after those 10 months using monthly interest rates of 10%, 1%, 0.5%, and 0.01%. Explain what you observe. **Exercise 10 is relevant to this topic.**

42. Reasonable balance? You are discussing a regular monthly deposit plan with an investment planner. The interest rate is fixed over the relatively short period of the investment. The planner tells you what ending balance you can expect. Explain how you can quickly check whether the figure the planner gave you is reasonable. **Exercise 12 is relevant to this topic.**

43. Your son's college education. You want to save money to finance your young son's college education. Explain the financial implications of waiting five more years to start saving for this expense. Explain your reasoning. **Exercise 18 is relevant to this topic.**

44. Starting early. Financial advisors counsel young people to start saving early for their retirement. Explain why an early start is important. **Exercise 31 is relevant to this topic.**

45. Reasonable annuity yield? You are discussing a 20-year annuity with an investment planner. The interest rate is fixed and relatively high. The planner tells you what monthly yield you can expect for the nest egg you have. Explain how you can quickly check whether the figure the planner gave you is reasonable. (Recall that annuities work just like the installment loans considered in Section 4.2.)

46. Annuities, plus and minus. Explain some of the advantages and disadvantages of a fixed-term annuity.

47. Perpetuities, plus and minus. *See Exercise 24 for background on perpetuities.* Explain some advantages and disadvantages of a perpetuity. **Exercise 27 is relevant to this topic.**

48. Fixed vs. life. Explain in your own words the difference between a fixed-term annuity and a life annuity.

4.4 Credit cards: Paying off consumer debt

TAKE AWAY FROM THIS SECTION
Be savvy in paying off credit cards.

The accompanying excerpt is from an article in *The Chronicle of Higher Education*. The subject might be familiar to you.

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The Hidden Portion of Student Loan Debt

LANCE LAMBERT May 8, 2015

More than six years after the 2008 financial crisis, American families have reduced household debt by about $900 billion. But one type of debt has been difficult to clear: student loans. That debt continued to grow during and after the downturn, and is now greater than both auto-loan and credit-card debt.

As of the end of 2014, outstanding student-loan debt topped $1.3 trillion. About $1.1 trillion of the total came from federal student-loan programs; the remainder was from private lenders.

Those figures, however, don't include other means of financing a college education. For example, students whose parents take out home-equity loans, or students who use credit cards to foot tuition bills, are not included in the student-loan-debt total.

⋮

In addition to the increased risks of alternate debt are increased costs. Students swipe their credit cards to pay not just for textbooks but also, in many cases, for tuition.

A 2014 survey by creditcards.com found that more than 260 colleges accepted credit-card payments for tuition–for a price. Those colleges typically charged a 2.6-percent convenience fee for the transaction. For a student paying a $10,000 tuition bill, that would add $260.

And, of course, students who don't pay their credit-card bills in full each month could end up paying hundreds or thousands of dollars more in interest.

Lambert, Lance, "The hidden portion of student loan debt," The Chronicle of Higher Education, 05/08/2015

Credit cards are convenient and useful. They allow us to travel without carrying large sums of cash, and they sometimes allow us to defer cash payments, even interest-free, for a short time. In fact, at times, having a credit card can be almost a necessity because many hotels and rental car companies require customers to have one.

Although credit cards are convenient, they come with a potential cost. For example, many credit cards carry an APR that is much higher than other kinds of consumer loans. At the time of this writing, the site creditcards.com showed APRs on credit cards generally ranging from about 18% to almost 24% for those with poor credit. (It showed a rate of 15% for student cards.) Credit cards can differ in other ways as well. In deciding which card to use, it is very important to read the fine print and get the card that is most favorable to you.

In this section, we explore credit cards and finance charges in some detail. In particular, we look at how payments are calculated, the terminology used by credit card companies, and the implications of making only the minimum required payments.

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"I found the problem. We earn money 5 days a week, but we spend money 7 days a week."

Credit card basics

All credit cards have finance charges, but most companies will waive them if you pay the full amount owed each month (see **Figure 4.12**). When finance charges are incurred, different companies calculate them in different ways. They typically explain the method they use on the monthly statement. Three methods that are used are the *previous balance method*, the *average daily balance method*, and the *adjusted balance method*.

Bill Smith
Member Since 2001
Card number ending in: 1111
Billing Period: 01/23/21-02/24/21

Go Paperless! Click Here

How to reach us
www.citicards.com
1-888-766-2484
BOX 6500 SIOUX FALLS, SD 57117

Minimum payment due:	$115.00
New balance:	$7,720.17
Payment due date:	03/20/21

Make a payment now! www.payonline.citicards.com

Late Payment Warning: If we do not receive your minimum payment by the date listed above, you may have to pay a late fee of up to $35 and your APRs may be increased up to the variable Penalty APR of 29.99%.

For information about credit counseling services, call 1-877-337-8187

Account Summary	
Previous balance	$7,114.54
Payments	–$7,114.54
Credits	–$52.79
Purchases	+$7,772.96
Cash advances	+$0.00
Fees	+$0.00
Interest	+$0.00
New balance	**$7,720.17**

Credit Limit	
Revolving Credit limit Includes $12,900 cash advance limit	$41,000
Available Revolving credit Includes $12,900 available for cash advances	$33,279

FIGURE 4.12 A credit card statement. Look at the fine print!

In the previous balance method, the company calculates finance charges based on the balance at the end of the previous payment period.

In the average daily balance method, the company calculates finance charges based on the average of the daily balances over the payment period (usually one month). This means that a pair of jeans purchased early in the month will incur more finance charges than the same pair purchased late in the month. See Exercises 28 and 29 for more information on this method.

The adjusted balance method is perhaps the most common method, and it is the one we will use in this text. It starts with the balance from the previous month, subtracts payments you've made since the previous statement, and adds any new purchases. This is the amount that is subject to finance charges at the end of the month, assuming that the previous balance wasn't paid in full. After this finance charge has been added, we get the new balance. This procedure is summarized in the formula:

$$\text{New balance} = \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge}$$

The finance charge is calculated by applying the monthly interest rate (the APR divided by 12) to the amount subject to finance charges. Thus,

$$\text{Finance charge} = \frac{\text{APR}}{12} \times (\text{Previous balance} - \text{Payments} + \text{Purchases})$$

Let's look at a card that charges 21.6% APR. Suppose your previous statement showed a balance of \$300, you made a payment of \$100 in response to your previous statement, and you have new purchases of \$50.

Because the APR is 21.6%, we find the monthly interest rate using

$$\text{Monthly interest rate} = \frac{\text{APR}}{12} = \frac{21.6\%}{12} = 1.8\%$$

Now we calculate the finance charge:

$$\begin{aligned}\text{Finance charge} &= 0.018 \times (\text{Previous balance} - \text{Payments} + \text{Purchases}) \\ &= 0.018 \times (\$300 - \$100 + \$50) \\ &= 0.018 \times \$250 = \$4.50\end{aligned}$$

This makes your new balance

$$\begin{aligned}\text{New balance} &= \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge} \\ &= \$300 - \$100 + \$50 + \$4.50 = \$254.50\end{aligned}$$

Your new balance is \$254.50.

EXAMPLE 4.26 Calculating finance charges: Buying clothes

Suppose your Visa card calculates finance charges using an APR of 22.8%. Your previous statement showed a balance of \$500, in response to which you made a payment of \$200. You then bought \$400 worth of clothes, which you charged to your Visa card. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					

SOLUTION

We start by entering the information given.

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1	\$500.00	\$200.00	\$400.00		

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The APR is 22.8%, and we divide by 12 to get a monthly rate of 1.9%. In decimal form, this calculation is $0.228/12 = 0.019$. Therefore,

$$\begin{aligned}\text{Finance charge} &= 0.019 \times (\text{Previous balance} - \text{Payments} + \text{Purchases})\\ &= 0.019 \times (\$500 - \$200 + \$400)\\ &= 0.019 \times \$700 = \$13.30\end{aligned}$$

That gives a new balance of

$$\begin{aligned}\text{New balance} &= \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge}\\ &= \$500 - \$200 + \$400 + \$13.30 = \$713.30\end{aligned}$$

Your new balance is $713.30. Here is the completed table.

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1	$500.00	$200.00	$400.00	1.9% of $700 = $13.30	$713.30

TRY IT YOURSELF 4.26

This is a continuation of Example 4.26. You make a payment of $300 to reduce the $713.30 balance and then charge a TV costing $700. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1	$500.00	$200.00	$400.00	$13.30	$713.30
Month 2					

The answer is provided at the end of this section.

SUMMARY 4.7
Credit Card Basics

The formula for finding the finance charge is

$$\text{Finance charge} = \frac{\text{APR}}{12} \times (\text{Previous balance} - \text{Payments} + \text{Purchases})$$

The new balance is found by using the formula

$$\text{New balance} = \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge}$$

Making only minimum payments

Most credit cards require a minimum monthly payment, which is usually calculated as a fixed percentage of your balance or some fixed amount, whichever is more. For example, the minimum monthly payment might be 5% of the balance or $25, whichever is more. To keep things simple, we will assume in this section that the minimum monthly payment is a fixed percentage of your balance. We will see that if you make only this minimum payment, your balance will decrease very slowly and will follow an exponential pattern.

Nick M Do/Photodisc/Getty Images

Minimal payments reduce balances minimally.

EXAMPLE 4.27 Finding next month's minimum payment: One payment

We have a card with an APR of 24%. The minimum payment is 5% of the balance. Suppose we have a balance of \$400 on the card. We decide to stop charging and to pay it off by making the minimum payment each month. Calculate the new balance after we have made our first minimum payment and then calculate the minimum payment due for the next month.

SOLUTION

The first minimum payment is

$$\text{Minimum payment} = 5\% \text{ of balance} = 0.05 \times \$400 = \$20$$

The monthly interest rate is the APR divided by 12. In decimal form, this is $0.24/12 = 0.02$. Therefore, the finance charge is

$$\begin{aligned}\text{Finance charge} &= 0.02 \times (\text{Previous balance} - \text{Payments} + \text{Purchases})\\ &= 0.02 \times (\$400 - \$20 + \$0)\\ &= 0.02 \times \$380 = \$7.60\end{aligned}$$

That makes a new balance of

$$\begin{aligned}\text{New balance} &= \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge}\\ &= \$400 - \$20 + \$0 + \$7.60 = \$387.60\end{aligned}$$

The next minimum payment will be 5% of this:

$$\text{Minimum payment} = 5\% \text{ of balance} = 0.05 \times \$387.60 = \$19.38$$

TRY IT YOURSELF 4.27

This is a continuation of Example 4.27. Calculate the new balance after we have made our second minimum payment and then calculate the minimum payment due for the next month.

The answer is provided at the end of this section.

Let's pursue the situation described in the previous example. The following table covers the first four payments if we continue making the minimum 5% payment each month:

FIGURE 4.13 The balance from minimum payments is a decreasing exponential function.

Month	Previous balance	Minimum payment	Purchases	Finance charge	New balance
1	\$400.00	5% of \$400.00 = \$20.00	\$0.00	2% of \$380.00 = \$7.60	\$387.60
2	\$387.60	5% of \$387.60 = \$19.38	\$0.00	2% of \$368.22 = \$7.36	\$375.58
3	\$375.58	5% of \$375.58 = \$18.78	\$0.00	2% of \$356.80 = \$7.14	\$363.94
4	\$363.94	5% of \$363.94 = \$18.20	\$0.00	2% of \$345.74 = \$6.91	\$352.65

The table shows that the \$400 is not being paid off very quickly. In fact, after four payments, the decrease in the balance is only about \$47 (from \$400 to \$352.65). If we look closely at the table, we will see that the balances follow a pattern. To find the pattern, let's look at how the balance changes in terms of percentages:

Month 1: The new balance of \$387.60 is 96.9% of the initial balance of \$400.
Month 2: The new balance of \$375.58 is 96.9% of the Month 1 new balance of \$387.60.
Month 3: The new balance of \$363.94 is 96.9% of the Month 2 new balance of \$375.58.
Month 4: The new balance of \$352.65 is 96.9% of the Month 3 new balance of \$363.94.

The balance exhibits a constant percentage change. This makes sense: Each month, the balance is decreased by a constant percentage due to the minimum payment and increased by a constant percentage due to the finance charge. This pattern indicates that the balance is a decreasing exponential function. That conclusion is supported by the graph of the balance shown in **Figure 4.13**, which has the classic shape of exponential decay.

Because of this exponential pattern, we can find a formula for the balance on our credit card in the situation where we stop charging and pay off the balance by making the minimum payment each month.

FORMULA 4.12 Minimum Payment Balance Formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance} \times ((1 + r)(1 - m))^t$$

where r is the monthly interest rate and m is the minimum monthly payment as a percent of the balance. Both r and m are in decimal form. You should not round the product $(1 + r)(1 - m)$ when performing the calculation.

We provide a derivation of this formula in Algebraic Spotlight 4.5 at the end of this section.

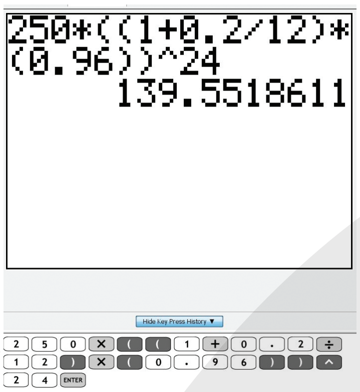
EXAMPLE 4.28 Using the minimum payment balance formula: Balance after two years

We have a card with an APR of 20% and a minimum payment that is 4% of the balance. We have a balance of $250 on the card, and we stop charging and pay off the balance by making the minimum payment each month. Find the balance after two years of payments.

SOLUTION

The APR in decimal form is 0.2, so the monthly interest rate in decimal form is $r = 0.2/12$. To avoid rounding, we leave r in this form. The minimum payment is 4% of the new balance, so we use $m = 0.04$. The initial balance is $250. The number of payments for two years is $t = 24$. Using the minimum payment balance formula (Formula 4.12), we find

$$\begin{aligned}\text{Balance after 24 minimum payments} &= \text{Initial balance} \times ((1+r)(1-m))^t \\ &= \$250 \times ((1+0.2/12)(1-0.04))^{24} \\ &= \$250 \times ((1+0.2/12)(0.96))^{24} \\ &= \$139.55\end{aligned}$$



Benny Evans

TRY IT YOURSELF 4.28

We have a card with an APR of 22% and a minimum payment that is 5% of the balance. We have a balance of $750 on the card, and we stop charging and pay off the balance by making the minimum payment each month. Find the balance after three years of payments.

The answer is provided at the end of this section.

We already noted that the credit card balance is not paid off quickly when we make only the minimum payment each month. The reason for this is now clear: The balance is a decreasing exponential function, and such functions typically decrease very slowly in the long run. The next example illustrates the dangers of making only the minimum monthly payment.

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Wise credit card use avoids debt.

EXAMPLE 4.29 Paying off your credit card balance: Long repayment

Suppose you have a balance of $10,000 on your Visa card, which has an APR of 24%. The card requires a minimum payment of 5% of the balance. You stop charging and begin making only the minimum payment until your balance is below $100.

a. Find a formula that gives your balance after t monthly payments.

b. Find your balance after five years of payments.

c. Determine how long it will take to get your balance under $100.[12]

d. Suppose that instead of the minimum payment, you want to make a fixed monthly payment so that your debt is clear in two years. How much do you pay each month?

---

[12] *It is worth noting that paying in this fashion will never actually get your balance to exactly zero. As mentioned earlier, however, the minimum monthly payment is usually stated in the form "5% of the balance or $25, whichever is larger." This method will ensure that the balance does get to zero.*

SOLUTION

a. The minimum payment as a decimal is $m = 0.05$, and the monthly rate as a decimal is $r = 0.24/12 = 0.02$. The initial balance is \$10,000. Using the minimum payment balance formula (Formula 4.12), we find

$$\begin{aligned}\text{Balance after } t \text{ minimum payments} &= \text{Initial balance} \times ((1+r)(1-m))^t \\ &= \$10{,}000 \times ((1+0.02)(1-0.05))^t \\ &= \$10{,}000 \times 0.969^t\end{aligned}$$

b. Now five years is 60 months, so we put $t = 60$ into the formula from part a:

$$\text{Balance after 60 months} = 10{,}000 \times 0.969^{60} = \$1511.56$$

After five years, we still owe more than \$1500.

c. We will show two ways to determine how long it takes to get the balance down to \$100.

Method 1: Using a logarithm: We need to solve for t the equation

$$\$100 = \$10{,}000 \times 0.969^t$$

The first step is to divide each side of the equation by \$10,000:

$$\begin{aligned}\frac{100}{10{,}000} &= \frac{10{,}000}{10{,}000} \times 0.969^t \\ 0.01 &= 0.969^t\end{aligned}$$

In Section 3.3, we learned how to solve exponential equations using logarithms. Summary 3.9 tells us that the solution for t of the equation $A = B^t$ is $t = \dfrac{\log A}{\log B}$. Using this formula with $A = 0.01$ and $B = 0.969$ gives

$$t = \frac{\log 0.01}{\log 0.969} \text{ months}$$

This is about 146.2 months. Hence, the balance will be under \$100 after 147 monthly payments, or more than 12 years of payments.

Method 2: Trial and error: If you want to avoid logarithms, you can solve this problem using trial and error with a calculator. The information in part b indicates that it will take some time for the balance to drop below \$100. So we might try 10 years, or 120 months. Computation using the formula from part a shows that after 10 years, the balance is still over \$200, so we should try a larger number of months. If we continue in this way, we find the same answer as that obtained from Method 1: The balance drops below \$100 at payment 147, which represents over 12 years of payments. (Spreadsheets and many calculators will create tables of values that make problems of this sort easy to solve.)

d. Making fixed monthly payments to clear your debt is like considering your debt as an installment loan: Just find the monthly payment if you borrow \$10,000 to buy (say) a car at an APR of 24% and pay the loan off over 24 months. We use the monthly payment formula from Section 4.2:

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

Recall that t is the number of months taken to pay off the loan, in this case 24, and that r is the monthly rate as a decimal, which in this case is 0.02.

Hence,

$$\text{Monthly payment} = \frac{\$10{,}000 \times 0.02 \times 1.02^{24}}{(1.02^{24} - 1)} = \$528.71$$

So a payment of \$528.71 each month will clear the debt in two years.

SUMMARY 4.8
Making Minimum Payments

Suppose we have a balance on our credit card and decide to stop charging and pay off the balance by making the minimum payment each month.

1. The balance is given by the exponential formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance} \times ((1 + r)(1 - m))^t.$$

In this formula, r is the monthly interest rate and m is the minimum monthly payment as a percent of the balance. Both r and m are in decimal form.

2. The product $(1 + r)(1 - m)$ should not be rounded when the calculation is performed.

3. Because the balance is a decreasing exponential function, the balance decreases very slowly in the long run.

Further complications of credit card usage

Situations involving credit cards can often be even more complicated than those discussed in previous examples in this section. For instance, in each of those examples, there was a single purchase and no further usage of the card. Of course, it's more common to make purchases every month. We also assumed that your payments were made on time, but there are substantial penalties for late or missed payments.

Use credit carefully.

Another complication is that credit card companies sometimes have "specials" or promotions in which you are allowed to skip a payment. And then there are *cash advances*, which are treated differently from purchases. Typically, cash advances incur finance charges immediately rather than after a month; that is, a cash advance is treated as carrying a balance immediately. Also, cash advances incur higher finance charges than purchases do.

Another complication occurs when your credit limit is reached (popularly known as "maxing out your card"). The credit limit is the maximum balance the credit card company allows you to carry. Usually, the limit is based on your credit history and your ability to pay. When you max out a credit card, the company will sometimes raise your credit limit, if you have always made required payments by its due dates.

In addition to all these complications, there are often devious hidden fees and charges. President Barack Obama signed into law the Credit Card Accountability Responsibility and Disclosure (CARD) Act of 2009. Among other things, the law requires that credit card companies inform cardholders how long it will take to pay off the balance if they make only the minimum payment. The following excerpt is from an article at the Web site of the federal Consumer Financial Protection Bureau.

IN THE NEWS 4.8

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CFPB Finds CARD Act Helped Consumers Avoid More Than $16 Billion in Gotcha Credit Card Fees

Concerns Remain About Other Back-End Practices Such As Deferred-Interest Promotions

CONSUMER FINANCIAL PROTECTION BUREAU December 3, 2015

Washington, D.C. The Consumer Financial Protection Bureau (CFPB) today released a report detailing how the Credit Card Accountability Responsibility and Disclosure Act (CARD Act) has helped reduce the cost of "gotcha" credit card fees by more than $16 billion. Since the reform law, total costs to consumers have fallen with the elimination of certain back-end pricing practices such as over-limit fees. Over the same period, credit has generally become more available to consumers and the number of new accounts has grown faster than in almost every other major consumer credit market. Concerns remain, however, about other back-end practices such as deferred-interest promotions that can hit consumers with unexpected costs.

"The CARD Act has helped people avoid more than $16 billion in gotcha credit card fees," said CFPB Director Richard Cordray. "The law made it easier for consumers to evaluate costs and risks by eliminating the worst back-end pricing practices in the market. There is more work to do. But with commonsense rules in place, credit cards are safer and more affordable, credit is more available, and companies remain profitable with improved customer satisfaction."

More than 60 percent of adults own at least one credit card account. In the first six months of 2015, more than 14.5 billion credit card transactions accounted for more than $1.4 trillion in purchase volume. Before the CARD Act, widespread back-end pricing practices racked up costs for consumers through hidden fees and other gotchas. Signed into law in May 2009, the CARD Act created a fairer and more transparent market by protecting consumers against unexpected interest rate hikes, excessive late fees, and hard-to-avoid over-limit fees. The CFPB assumed authority for the CARD Act in July 2011.

The CARD Act directs the CFPB to regularly review the credit card market and the impact of the law's rules. This is the CFPB's second report. Today's report finds that, generally, consumers are paying less for their credit cards than they did before the law, and those costs are easier to predict before they are incurred. In addition, credit availability has continued to expand for consumers.

ALGEBRAIC SPOTLIGHT 4.5 Derivation of the Minimum Payment Balance Formula

Suppose a credit card has an initial balance. Assume that we incur no further charges and make only the minimum payment each month. Suppose r is the monthly interest rate and m is the minimum monthly payment as a percentage of the new balance. Both r and m are in decimal form. Our goal is to derive the formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance} \times ((1+r)(1-m))^t$$

Here is the derivation. Assume that we have made a series of payments, and let B denote the balance remaining. We need to calculate the new balance. First we find the minimum payment on the balance B. To do so, we multiply B by m:

$$\text{Minimum payment} = mB$$

Next we find the amount subject to finance charges:

$$\begin{aligned}\text{Amount subject to finance charges} &= \text{Previous balance} - \text{Payments}\\ &= B - mB\\ &= B(1-m)\end{aligned}$$

To calculate the finance charge, we apply the monthly rate r to this amount:

$$\text{Finance charge} = r \times B(1-m)$$

Therefore, the new balance is

$$\begin{aligned}\text{New balance} &= \text{Previous balance} - \text{Payments} + \text{Finance charge}\\ &= B(1-m) + rB(1-m)\\ &= B \times ((1-m) + r(1-m))\\ &= B \times ((1+r)(1-m))\end{aligned}$$

We can write this as

$$\text{New balance} = \text{Previous balance} \times ((1+r)(1-m))$$

Therefore, to find the new balance each month, we multiply the previous balance by $(1+r)(1-m)$. That makes the balance after t payments an exponential function of t with base $(1+r)(1-m)$. The initial value of this function is the initial balance, so we have the exponential formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance} \times ((1+r)(1-m))^t$$

This is the minimum payment balance formula.

WHAT DO YOU THINK?

Which credit card? You are shopping for a credit card. What features would you look for to help you decide which card to choose? You might look first at features discussed in this section.

Questions of this sort may invite many correct responses. We present one possibility: You certainly want to know the interest rate, but you also need to know how finance charges are calculated. The method of “average daily balance” is thought by some to be the fairest method of arriving at the balance on which charges are calculated. You also need to understand penalties charged by the card. They can be substantial and result in long-lasting problems. Credit cards are best used if the balance is paid in full each month, but in case that isn't possible, you need to know how the card deals with minimum payments. There are many more credit card issues that are important, including rewards programs. Recent laws are designed to limit predatory practices of credit card companies, but in the end it is up to you to "read the fine print." If you don't understand how a credit card works, don't get that card.

Further questions of this type may be found in the *What Do You Think?* section of exercises.

Try It Yourself answers

Try It Yourself 4.26: Calculating finance charges: Buying clothes

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1	\$500.00	\$200.00	\$400.00	\$13.30	\$713.30
Month 2	\$713.30	\$300.00	\$700.00	\$21.15	\$1134.45

Try It Yourself 4.27: Finding next month's minimum payment: One payment New balance: \$375.58; minimum payment: \$18.78.

Try It Yourself 4.28: Using the minimum payment balance formula: Balance after three years \$227.59.

Exercise Set 4.4

Test Your Understanding

1. True or false: Most credit card companies will waive finance charges if you pay the monthly balance in full.

2. Which of the following affect finance charges on a credit card? **a.** your previous month's balance **b.** your purchases this month **c.** the amount of your payment **d.** all of the above.

3. True or false: Putting rounded numbers into financial formulas can cause significant errors.

4. True or false: The APR on most credit cards is well over 10%.

5. Which of the following should you expect to see on a credit card statement? **a.** the amount of purchases for the month **b.** the amount of finance charges for the month **c.** the balance of your account for the month **d.** the balance of your account for last month **e.** all of these.

6. If you make the minimum monthly payment of 5% on your credit card with no further purchases, your balance will: **a.** decrease exponentially **b.** decrease linearly **c.** remain about the same.

7. True or false: If the minimum monthly payment on your credit card is 5% or \$25, whichever is larger, and if you make that payment each month, then with no further purchases your balance will eventually get to zero.

8. True or false: If you owe \$5000 on your credit card and make only the minimum monthly payment, then with no further purchases your balance will decrease rapidly.

Problems

9. Amount subject to finance charge. The previous statement for your credit card had a balance of \$540. You make purchases of \$150 and make a payment of \$60. The credit card has an APR of 22%. What is the finance charge for this month?

10. Calculating finance charge. The previous statement for your credit card had a balance of \$650. You make a payment of \$300 but you made additional purchases of \$200. The card carries an APR of 24%. What is the finance charge for this month?

11. Calculating minimum payment. You have a credit card with an APR of 36%. The minimum payment is 10% of the balance. Suppose you have a balance of \$800. You decide to stop charging and make only the minimum payment. The initial payment will be 10% of \$800, or \$80. Calculate the minimum payment for the next two months. (Be sure to take into account the finance charges.)

12. Using the minimum payment balance formula. You have a credit card with an APR of 24%. The card requires a minimum monthly payment of 5% of the balance. You have a balance of $7500. You stop charging and make only the minimum monthly payment. What is the balance on the card after five years?

13. Calculating balances. You have a credit card with an APR of 16%. You begin with a balance of $800. In the first month you make a payment of $400 and you make charges amounting to $300. In the second month you make a payment of $300 and you make new charges of $600. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					
Month 2					

14. Calculating balances. You have a credit card with an APR of 20%. You begin with a balance of $600. In the first month you make a payment of $400 and make new charges of $200. In the second month you make a payment of $300 and make new charges of $100. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					
Month 2					

15. Calculating balances. You have a credit card with an APR of 22.8%. You begin with a balance of $1000, in response to which you make a payment of $200. The first month you make charges amounting to $500. You make a payment of $200 to reduce the new balance, and the second month you charge $600. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					
Month 2					

16. Calculating balances. You have a credit card with an APR of 12%. You begin with a balance of $200, in response to which you make a payment of $75. The first month you make charges amounting to $50. You make a payment of $75 to reduce the new balance, and the second month you charge $60. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					
Month 2					

17. A balance statement. Assume that you start with a balance of $4500 on your Visa credit card. During the first month, you charge $500, and during the second month, you charge $300. Assume that Visa has finance charges of 24% APR and that each month you make only the minimum payment of 2.5% of the balance. Complete the following table:

Month	Previous balance	Minimum payment	Purchases	Finance charge	New balance
1	$4500.00				
2					

18. A balance statement. Assume that you start with a balance of $4500 on your MasterCard. Assume that MasterCard has finance charges of 12% APR and that each month you make only the minimum payment of 3% of the balance. During the first month, you charge $300, and during the second month, you charge $600. Complete the following table:

Month	Previous balance	Minimum payment	Purchases	Finance charge	New balance
1	$4500.00				
2					

19. New balances. Assume that you have a balance of $4500 on your Discover credit card and that you make no more charges. Assume that Discover charges 21% APR and that each month you make only the minimum payment of 2.5% of the balance.

a. Find a formula for the balance after t monthly payments.

b. What will the balance be after 30 months?

c. What will the balance be after 10 years?

20. Paying tuition on your American Express card at the maximum interest rate. You have a balance of $10,000 for your tuition on your American Express credit card. Assume that you make no more charges on the card. Also assume that American Express charges 24% APR and that each month you make only the minimum payment of 2% of the balance.

a. Find a formula for the balance after t monthly payments.

b. How much will you owe after 10 years of payments?

c. How much would you owe if you made 100 years of payments?

d. Find when the balance would be less than $50.

21. Paying tuition on your American Express card. You have a balance of $10,000 for your tuition on your American Express credit card. Assume that you make no more charges on the card. Also assume that American Express charges 12% APR and that each month you make only the minimum payment of 4% of the balance.

a. Find a formula for the balance after t monthly payments.

b. How long will it take to get the balance below $50?

22. Paying off a Visa card. You have a balance of $1000 on your Visa credit card. Assume that you make no more charges on the card and that the card charges 9.9% APR and requires a minimum payment of 3% of the balance. Assume also that you make only the minimum payments.

a. Find a formula for the balance after t monthly payments.

b. Find how many months it takes to bring the balance below \$50.

23. Balance below \$200. You have a balance of \$4000 on your credit card, and you make no more charges. Assume that the card requires a minimum payment of 5% and carries an APR of 22.8%. Assume also that you make only the minimum payments. Determine when the balance drops below \$200.

24. New balances. Assume that you have a balance of \$3000 on your MasterCard and that you make no more charges. Assume that MasterCard charges 12% APR and that each month you make only the minimum payment of 5% of the balance.

a. Find a formula for the balance after t monthly payments.

b. What will the balance be after 30 months?

c. What will the balance be after 10 years?

d. At what balance do you begin making payments of \$20 or less?

e. Find how many months it will take to bring the remaining balance down to the value from part d.

25. Paying off a Discover card. Assume that you have a balance of \$3000 on your Discover credit card and that you make no more charges. Assume that Discover charges 21% APR and that each month you make only the minimum payment of 2% of the balance.

a. Find a formula for the remaining balance after t monthly payments.

b. On what balance do you begin making payments of \$50 or less?

c. Find how many months it will take to bring the remaining balance down to the value from part b.

26. Monthly payment. You have a balance of \$400 on your credit card and make no more charges. Assume that the card carries an APR of 18%. Suppose you wish to pay off the card in six months by making equal payments each month. What is your monthly payment?

27. What can you afford to charge? Suppose you have a new credit card with 0% APR for a limited period. The card requires a minimum payment of 5% of the balance. You feel you can afford to pay no more than \$250 each month. How much can you afford to charge? How much could you afford to charge if the minimum payment were 2% instead of 5%?

28. Average daily balance. The most common way of calculating finance charges is not the simplified one we used in this section but rather the *average daily balance*. With this method, we calculate the account balance at the end of each day of the month and take the average. That average is the amount subject to finance charges. To simplify things, we assume that the billing period is one week rather than one month. Assume that the weekly rate is 1%. You begin with a balance of \$500. On day 1, you charge \$75. On day 3, you make a payment of \$200 and charge \$100. On day 6, you charge \$200.

a. Assume that finance charges are calculated using the simplified method shown in this section. Find the account balance at the end of the week.

b. Assume that finance charges are calculated using the average daily balance. Find the account balance at the end of the week.

29. More on average daily balance. The method for calculating finance charges based on the average daily balance is explained in the setting for Exercise 28. As in that exercise, to simplify things, we assume that the billing period is one week rather than one month and that the weekly rate is 1%. You begin with a balance of \$1000. On day 1, you charge \$200. On day 3, you charge \$500. On day 6, you make a payment of \$400 and you charge \$100.

a. Assume that finance charges are calculated using the simplified method shown in this section. Find the account balance at the end of the week.

b. Assume that finance charges are calculated using the average daily balance. Find the account balance at the end of the week.

30. Finance charges versus minimum payments. In all of the examples in this section, the monthly finance charge is always less than the minimum payment. In fact, this is always the case. Explain what would happen if the minimum payment were less than the monthly finance charge.

Exercises 31 and 32 are designed to be solved using technology such as calculators or computer spreadsheets.

31. Paying off a Visa card—in detail. Assume that you have a balance of \$1000 on your Visa credit card and that you make no more charges on the card. Assume that Visa charges 12% APR and that the minimum payment is 5% of the balance each month. Assume also that you make only the minimum payments. Make a spreadsheet listing the items below for each month until the payment falls below \$20.

Month	Previous balance	Minimum payment	Purchases	Finance charge	New balance
1	\$1000.00				
2					
and so on					

32. Paying off an American Express card. Assume that you have a balance of \$3000 on your American Express credit card and that you make no more charges on the card. Assume that American Express charges 20.5% APR and that the minimum payment is 2% of the balance each month. Assume also that you make only the minimum payments. Make a spreadsheet listing the items below for each month until the payment drops below \$50.

Month	Previous balance	Minimum payment	Purchases	Finance charge	New balance
1	\$3000.00				
2					
and so on					

Writing About Mathematics

33. Average daily balance. We noted that most credit cards use the average daily balance to assess finance charges, and this method was investigated in Exercises 28 and 29. Many regard this method as fair to all, but some do not. Do some research to form your own opinion on this issue and report your conclusions.

34. Location. Many credit card companies have operations in Sioux Falls, South Dakota. (In fact, if you look at Figure 4.12 you will see a reference to that city.) Do some research to find out why and write a report on your findings.

35. Rules. We mentioned at the outset that different credit card companies may have different rules. Investigate some companies to see whether you can detect any differences other than interest rates. Write a report on your findings.

36. Chips. Credit cards have had magnetic stripes for a long time, but recently they have also come with implanted computer chips. Investigate why chips are being used and write a report on your findings.

37. A bit of history. Credit cards are relatively new. Write a brief report on the introduction of credit cards into the U.S. economy.

What Do You Think?

38. Your own credit card. Examine the monthly statement from one of your own credit cards. Note the ways it is similar to examples in the text and also note differences. Write down any parts of the statement you do not understand. **Exercise 15 is relevant to this topic.**

39. What is the APR? Credit cards often advertise the monthly interest rate rather than the APR. Use this example to explain why they do so: Suppose a credit card company advertises an interest rate of 3% per month. What is the APR for this card? Which sounds more appealing, the monthly rate or the APR?

40. Understanding repayment. A friend owes \$10,000 on his credit card. He plans to stop making charges and pay off the balance. The card has a minimum payment of 5%. He figures correctly that 5% of \$10,000 is \$500. From that he concludes that he needs to make the 5% minimum payment for only 20 months, because $20 \times \$500 = \$10{,}000$. Is your friend right? If not, explain the error in his reasoning. **Exercise 20 is relevant to this topic.**

41. Purchasing a home. Why would it be unwise (even if it were allowed) to charge the purchase of a home to a credit card? Go beyond the credit limit to consider factors such as interest rates, budgeting, etc.

42. Slow repayment. In this section we looked at repaying the balance on your credit card by making the minimum payment each month. We saw that this method results in a decreasing exponential function for the amount owed. How is this situation analogous to the long-term problem of handling radioactive waste discussed in Section 3.2? How are the underlying mathematical formulas similar?

43. Company profit. Most credit card companies charge you nothing if you pay off your balance each month. Do some research on the Internet to determine how the company makes a profit from customers who pay off their balance each month.

44. Balance decreases. In the examples presented in this section, making the minimum monthly payment decreased the balance owed. Why would credit card companies want to be sure this decrease happens? Would this decrease happen if the APR were 24% and the minimum monthly payment were 1%? **Exercise 30 is relevant to this topic.**

45. Credit card rates. Credit cards sometimes come with very high interest rates. Use the Internet to investigate interest rates commonly associated with credit cards. (Try to find the interest rate associated with your own credit card.)

46. How much to pay. You have acquired an extra bit of money and you are trying to decide the best way to use it. You can pay down a credit card that has a high interest rate, or you can put the money into a savings account with a low interest rate and save it for a rainy day. Which would you choose and why? Are there additional options you might consider?

4.5 Inflation, taxes, and stocks: Managing your money

TAKE AWAY FROM THIS SECTION
Understand the impact of inflation and the significance of marginal tax rates.

The following is from the Web site of Bloomberg News.

IN THE NEWS 4.9

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The Fed Follows a Script, But Inflation Isn't Playing Its Part

Where are the wage increases that should go along with such low unemployment?

DANIEL MOSS July 05, 2017

The Federal Reserve still has faith, even if some economic numbers are moving the wrong way.

The minutes of Fed officials' June 13-14 meeting tell us something about the current soft patch in U.S. data, especially inflation.

The Fed is largely sticking to the script. The retreat in inflation is transitory, idiosyncratic even, and the slow-but-steady slog back toward the central bank's 2 percent target will probably resume. The overall tone is one of economic optimism, guarded in places, and helped by a better global picture. Gradual interest-rate increases can continue, with some debate about timing and magnitude. The lack of wage growth and inflation doesn't seem to shake most officials' confidence.

Why is this important? It's tied up with the question of when the very low unemployment rate will result in significant and consistent wage increases, arguably the most important missing ingredient since the economy began growing again in the second half of 2009. Without inflation, workers are less likely to agitate for higher pay. And without jumps in compensation, it's tougher for inflation to get back to a healthy level.

Granted, the labor market keeps strengthening, even if wages have been subdued. At 4.3 percent, the jobless rate keeps punching through the Fed's forecasts and is below the level officials consider is sustainable over the long run. At this point, the prevailing wisdom is that there is so little slack that wages and inflation just have to – at some point – really start to pick up.

Chris Ratcliffe/Bloomberg/Getty Images

Janet Yellen leads the faithful.

This article highlights the connection between inflation, wage growth, and interest rates in the growth of the economy. We will explore several related financial issues in this section.

CPI and the inflation rate

The preceding article states the concerns of the Federal Reserve Board about inflation. But what is inflation, and how is it measured?

Inflation is calculated using the *Consumer Price Index* (CPI), which is a measure of the price of a certain "market basket" of consumer goods and services relative to a predetermined benchmark. When the CPI goes up, we have inflation. When it goes down, we have *deflation*.

According to the U.S. Department of Labor, this "market basket" consists of commodities in the following categories:

- **Food and beverages** (breakfast cereal, milk, coffee, chicken, wine, service meals and snacks)
- **Housing** (rent of primary residence, owners' equivalent rent, fuel oil, bedroom furniture)
- **Apparel** (men's shirts and sweaters, women's dresses, jewelry)
- **Transportation** (new vehicles, airline fares, gasoline, motor vehicle insurance)

- **Medical care** (prescription drugs and medical supplies, physicians' services, eyeglasses and eye care, hospital services)
- **Recreation** (televisions, pets and pet products, sports equipment, admissions)
- **Education and communication** (college tuition, postage, telephone services, computer software and accessories)
- **Other goods and services** (tobacco and smoking products, haircuts and other personal services, funeral expenses)

KEY CONCEPT

The Consumer Price Index (CPI) is a measure of the average price paid by urban consumers for a "market basket" of consumer goods and services.

The *rate of inflation* is measured by the percentage change in the CPI.

KEY CONCEPT

An increase in prices is referred to as inflation. The rate of inflation is measured by the percentage change in the Consumer Price Index over time. When prices decrease, the percentage change is negative; this is referred to as deflation.

Inflation reflects a decline of the purchasing power of the consumer's dollar. Besides affecting how much we can afford to buy, the inflation rate has a big influence on certain government programs that affect the lives of many people.

For example, as of 2020, there were about 64.9 million Social Security beneficiaries and 39.9 million SNAP (food stamp) recipients who are affected by the CPI because their benefits are adjusted periodically to compensate for inflation. In addition, millions of military and federal civil service retirees and subsidized lunches at schools are affected.

Table 4.3 shows the annual change in prices in the United States over a 71-year period. For example, from December 1949 to December 1950, the CPI changed from 23.6 to 25.0, an increase of $25.0 - 23.6 = 1.4$. Now we find the percentage change:

$$\text{Percentage change} = \frac{\text{Change in CPI}}{\text{Previous CPI}} \times 100\% = \frac{1.4}{23.6} \times 100\%$$

This is about 5.9%, and that is the inflation rate for this period shown in the table. Usually, we round the inflation rate as a percentage to one decimal place.

EXAMPLE 4.30 Calculating inflation: CPI increase to 205

Suppose the CPI increases this year from 200 to 205. What is the rate of inflation for this year?

SOLUTION

The change in the CPI is $205 - 200 = 5$. To find the percentage change, we divide the increase of 5 by the original value of 200 and convert to a percent:

$$\text{Percentage change} = \frac{\text{Change in CPI}}{\text{Previous CPI}} \times 100\% = \frac{5}{200} \times 100\% = 2.5\%$$

Therefore, the rate of inflation is 2.5%.

TRY IT YOURSELF 4.30

Suppose the CPI increases this year from 215 to 225. What is the rate of inflation for this year?

The answer is provided at the end of this section.

TABLE 4.3 Historical Inflation

CPI in December and inflation rate					
December	CPI	Inflation rate	December	CPI	Inflation rate
1949	23.6	—	1985	109.3	3.8%
1950	25.0	5.9%	1986	110.5	1.1%
1951	26.5	6.0%	1987	115.4	4.4%
1952	26.7	0.8%	1988	120.5	4.4%
1953	26.9	0.7%	1989	126.1	4.6%
1954	26.7	−0.7%	1990	133.8	6.1%
1955	26.8	0.4%	1991	137.9	3.1%
1956	27.6	3.0%	1992	141.9	2.9%
1957	28.4	2.9%	1993	145.8	2.7%
1958	28.9	1.8%	1994	149.7	2.7%
1959	29.4	1.7%	1995	153.5	2.5%
1960	29.8	1.4%	1996	158.6	3.3%
1961	30.0	0.7%	1997	161.3	1.7%
1962	30.4	1.3%	1998	163.9	1.6%
1963	30.9	1.6%	1999	168.3	2.7%
1964	31.2	1.0%	2000	174.0	3.4%
1965	31.8	1.9%	2001	176.7	1.6%
1966	32.9	3.5%	2002	180.9	2.4%
1967	33.9	3.0%	2003	184.3	1.9%
1968	35.5	4.7%	2004	190.3	3.3%
1969	37.7	6.2%	2005	196.8	3.4%
1970	39.8	5.6%	2006	201.8	2.5%
1971	41.1	3.3%	2007	210.0	4.1%
1972	42.5	3.4%	2008	210.2	0.1%
1973	46.2	8.7%	2009	215.9	2.7%
1974	51.9	12.3%	2010	219.2	1.5%
1975	55.5	6.9%	2011	225.7	3.0%
1976	58.2	4.9%	2012	229.6	1.7%
1977	62.1	6.7%	2013	233.0	1.5%
1978	67.7	9.0%	2014	234.8	0.8%
1979	76.7	13.3%	2015	236.5	0.7%
1980	86.3	12.5%	2016	241.4	2.1%
1981	94.0	8.9%	2017	246.5	2.1%
1982	97.6	3.8%	2018	251.2	1.9%
1983	101.3	3.8%	2019	257.0	2.3%
1984	105.3	3.9%	2020	260.5	1.4%

cartoonstock.com

"When you take out food, energy, taxes, insurance, housing, transportation, healthcare, and entertainment, inflation remained at a 20 year low."

Table 4.4 lists four countries and their estimated annual rates of inflation for 2013. A very high rate of inflation, as in countries like Venezuela and Zimbabwe, is referred to as *hyperinflation*.

If the rate of inflation is 10%, one may think that the buying power of a dollar has decreased by 10%, but that is not the case. Inflation is the percentage change in prices, and that is not the same as the percentage change in the value of a dollar. To see the difference, let's imagine a frightening inflation rate of 100% this year. With such a rate, an item that costs \$200 this year will cost \$400 next year. This means that my money can buy only half as much next year as it can this year. So the buying power of a dollar would decrease by 50%, not by 100%.

TABLE 4.4 Examples of Hyperinflation in 2020

Country	Estimated inflation rate
Venezuela	6500%
Zimbabwe	623%
Sudan	142%
Lebanon	85%

The following formula tells us how much the buying power of currency decreases for a given inflation rate.

FORMULA 4.13 Buying Power Formula

$$\text{Percent decrease in buying power} = \frac{100\,i}{100 + i}$$

Here i is the inflation rate expressed as a percent, not a decimal. Usually we round the decrease in buying power as a percentage to one decimal place.

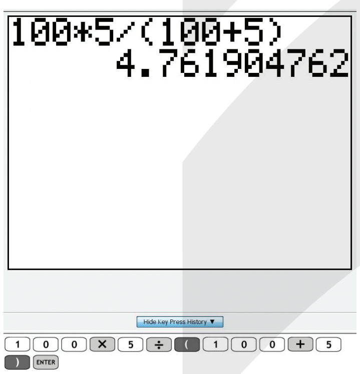
The buying power formula is derived in Algebraic Spotlight 4.6 at the end of this section.

EXAMPLE 4.31 Calculating decrease in buying power: 5% inflation

Suppose the rate of inflation this year is 5%. What is the percentage decrease in the buying power of a dollar?

SOLUTION

We use the buying power formula (Formula 4.13) with $i = 5\%$:



Benny Evans

$$\begin{aligned}\text{Percent decrease in buying power} &= \frac{100\,i}{100 + i}\\ &= \frac{100 \times 5}{100 + 5}\end{aligned}$$

This is about 4.8%.

TRY IT YOURSELF 4.31

According to Table 4.4, in 2020 the rate of inflation for Venezuela was 6500%. What was the percentage decrease that year in the buying power of the *bolívar* (the currency of Venezuela)?

The answer is provided at the end of this section.

A companion formula to the buying power formula (Formula 4.13) gives the inflation rate in terms of the percent decrease in buying power of currency.

FORMULA 4.14 Inflation Formula

$$\text{Percent rate of inflation} = \frac{100B}{100 - B}$$

In this formula, B is the decrease in buying power expressed as a percent, not as a decimal.

EXAMPLE 4.32 Calculating inflation: 2.5% decrease in buying power

Suppose the buying power of a dollar decreases by 2.5% this year. What is the rate of inflation this year?

SOLUTION

We use the inflation formula (Formula 4.14) with $B = 2.5\%$:

$$\begin{aligned}\text{Percent rate of inflation} &= \frac{100B}{100 - B}\\ &= \frac{100 \times 2.5}{100 - 2.5}\end{aligned}$$

This is about 2.6%.

TRY IT YOURSELF 4.32

Suppose the buying power of a dollar decreases by 5.2% this year. What is the rate of inflation this year?

The answer is provided at the end of this section.

The next example covers all of the concepts we have considered so far in this section.

EXAMPLE 4.33 Understanding inflation and buying power: Effects on goods

Parts a through c refer to Table 4.3.

a. Find the 10-year inflation rate in the United States from December 2010 to December 2020.

b. If a sofa cost \$100 in December 2008 and the price changed in accordance with the inflation rate in the table, how much did the sofa cost in December 2009?

c. If a chair cost \$50 in December 1953 and the price changed in accordance with the inflation rate in the table, how much did the chair cost in December 1954?

d. According to Table 4.4, in 2020 the rate of inflation for Lebanon was 85%. How much did the buying power of the currency, the *lira*, decrease during the year?

e. In Ethiopia, the buying power of the currency, the *birr*, decreased by 16.8% in 2020. What was Ethiopia's inflation rate for 2020?

SOLUTION

a. In 2010 the CPI was 219.2, and in 2020 it was 260.5. The increase was $260.5 - 219.2 = 41.3$, so

$$\text{Percentage change} = \frac{\text{Change in CPI}}{\text{Previous CPI}} \times 100\% = \frac{41.3}{219.2} \times 100\%$$

or about 18.8%. The 10-year inflation rate was about 18.8%.

b. According to the table, the inflation rate was 2.7% in 2009, which tells us that the price of the sofa increased by 2.7% during that year. Therefore, it cost $\$100 + 0.027 \times \$100 = \$102.70$ in December 2009.

c. The inflation rate is -0.7%, which tells us that the price of the chair decreased by 0.7% during that year. Because 0.7% of \$50.00 is \$0.35, in December 1954 the chair cost $\$50.00 - \$0.35 = \$49.65$.

d. We use the buying power formula (Formula 4.13) with $i = 85\%$:

$$\begin{aligned}\text{Percent decrease in buying power} &= \frac{100\,i}{100 + i}\\ &= \frac{100 \times 85}{100 + 85}\end{aligned}$$

or about 45.9%. The buying power of the Lebanese *lira* decreased by 45.9%.

e. We want to find the inflation rate for Ethiopia. We know that the reduction in buying power was 16.8%, so we use the inflation formula (Formula 4.14) with $B = 16.8\%$:

$$\begin{aligned}\text{Percent rate of inflation} &= \frac{100B}{100 - B}\\ &= \frac{100 \times 16.8}{100 - 16.8}\end{aligned}$$

or about 20.2%. Therefore, the rate of inflation in Ethiopia in 2020 was about 20.2%.

SUMMARY 4.9
Inflation and Reduction of Currency Buying Power

If the inflation rate is i (expressed as a percent), the change in the buying power of currency can be calculated using

$$\text{Percent decrease in buying power} = \frac{100\,i}{100 + i}$$

A companion formula gives the inflation rate in terms of the decrease B (expressed as a percent) in buying power:

$$\text{Percent rate of inflation} = \frac{100B}{100 - B}$$

Income taxes

Consider the following tax tables for the year 2020 from the Internal Revenue Service. **Table 4.5** shows tax rates for single people, and **Table 4.6** shows tax rates for married couples filing jointly. Note that the tax rates are applied to *taxable income*. The percentages in the tables are called *marginal rates*, and they apply only to earnings in excess of a certain amount. With a marginal tax rate of 12%, for example, the tax owed increases by $0.12 for every $1 increase in taxable income. This makes the tax owed a linear function of the taxable income within a given range of incomes. The slope is the marginal tax rate (as a decimal).

TABLE 4.5 2020 Tax Table for Singles

If Taxable Income		The Tax is		
Is over	But not over	This amount	Plus this %	Of the excess over
Schedule X—Use if your filing status is Single				
$0	$9875	—-	10 %	$0
9876	40,125	$987.50	12 %	9875
40,126	85,525	4617.50	22 %	40,125
85,526	163,300	14,605.50	24 %	85,525
163,301	207,350	33,271.50	32 %	163,300
207,351	518,400	47,367.50	35 %	207,350
518,401	—–	156,235.00	37 %	518,400

TABLE 4.6 2020 Tax Table for Married Couples Filing Jointly

If Taxable Income		The Tax is		
Is over	But not over	This amount	Plus this %	Of the excess over
Schedule Y-1—Use if your filing status is Married filing jointly or Qualifying widow(er)				
$0	$19,750	—-	10 %	$0
19,751	80,250	$1975.00	12 %	19,750
80,251	171,050	9235.00	22 %	80,250
171,051	326,600	29,211.00	24 %	171,050
326,601	414,700	66,543.00	32 %	326,600
414,701	622,050	94,735.00	35 %	414,700
622,051	—–	167,307.50	37 %	622,050

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Income taxes are part of financial planning.

Note that these marginal rates increase as you earn more and so move from one *tax bracket* to another. A system of taxation in which the marginal tax rates increase for higher incomes is referred to as a *progressive tax*.[13]

EXAMPLE 4.34 Calculating the tax: A single person

In the year 2020, Alex was single and had a taxable income of $70,000. How much tax did she owe?

SOLUTION

According to Table 4.5, Alex owed $4617.50 plus 22% of the excess taxable income over $40,125. The total tax is

$$\$4617.50 + 0.22 \times (\$70{,}000 - \$40{,}125) = \$11{,}190.00$$

TRY IT YOURSELF 4.34

In the year 2020, Bob was single and had a taxable income of $20,000. How much tax did he owe?

The answer is provided at the end of this section.

A person's taxable income is obtained by subtracting certain *deductions* from total income. Deductions may include things like state and local taxes, home mortgage interest, and charitable contributions. Some people do not have very many of these kinds of deductions. In this case, they may choose not to itemize them but rather to take what is called a "standard deduction."

EXAMPLE 4.35 Comparing taxes: The "marriage penalty"

a. In the year 2020, Ann was single. After all deductions her taxable income was $320,000. How much income tax did she owe?

b. In the year 2020, Alice and Bob were married. Together they had a taxable income of $640,000 per year and filed jointly. How much income tax did they owe?

[13] *In 1996 and 2000, magazine publisher Steve Forbes ran for president on a platform that included a 17% "flat tax." (The flat-tax concept is explored a bit more in the exercises at the end of this section.) In that system everyone would pay 17% of their taxable income no matter what that income is. In a speech Forbes said, "When we're through with Washington, the initials of the IRS will be RIP." Forbes did not win the Republican nomination.*

c. Explain what you notice about the amount of tax paid in the two situations in parts a and b.

SOLUTION

a. According to the tax table for singles in Table 4.5, Ann owed $47,367.50 plus 35% of everything over $207,350, which is $86,795.00 in income tax.

b. According to the tax table for married couples shown in Table 4.6, Alice and Bob owe $167,307.50 plus 37% of everything over $622,050, which comes to $173,949.00.

c. If Alice and Bob each had a taxable income of $320,000 and could file as singles, then, using the solution to part a, they would owe 2 × $86,795.00 = $173,590.00. But because they were married and filing jointly, they had to pay $173,949.00, which is an extra $359 in taxes.

Marty Bucella/cartoonstock.com

"We were going over some of your returns from a past life and..."

The disparity in part c of Example 4.35 is referred to as the "marriage penalty." Many people believe the marriage penalty is unfair, but there are those who argue that it makes sense to tax a married couple more than two single people. Can you think of some arguments to support the two sides of this question? The marriage penalty has been eliminated for most middle-income earners.

Claiming deductions lowers your tax by reducing your taxable income. Another way to lower your tax is to take a *tax credit*. Here is how to apply a tax credit: Calculate the tax owed using the tax tables (making sure first to subtract from the total income any deductions) and then subtract the tax credit from the tax determined by the tables. Because a tax credit is subtracted directly from the tax you owe, a tax credit of $1000 has a much bigger impact on lowering your taxes than a deduction of $1000. That is the point of the following example.

EXAMPLE 4.36 Comparing deductions and credits: Differing effects

In the year 2020, Betty and Carol were single, and each had a total income of $80,000. Betty took a deduction of $15,000 but had no tax credits. Carol took a deduction of $14,000 and had an education tax credit of $1000. Compare the tax owed by Betty and Carol.

SOLUTION

The taxable income of Betty is $\$80{,}000 - \$15{,}000 = \$65{,}000$. According to the tax table in Table 4.5, Betty owes \$4617.50 plus 22% of the excess taxable income over \$40,125. That tax is

$$\$4617.50 + 0.22 \times (\$65{,}000 - \$40{,}125) = \$10{,}090$$

Betty has no tax credits, so the tax she owes is \$10,090.

The taxable income of Carol is $\$80{,}000 - \$14{,}000 = \$66{,}000$. According to the tax table in Table 4.5, before applying tax credits, Carol owes \$4617.50 plus 22% of the excess taxable income over \$40,125. That tax is

$$\$4617.50 + 0.22 \times (\$66{,}000 - \$40{,}125) = \$10{,}310$$

Carol has a tax credit of \$1000, so the tax she owes is

$$\$10{,}310 - \$1000 = \$9310$$

Betty owes

$$\$10{,}090 - \$9310 = \$780$$

more tax than Carol.

TRY IT YOURSELF 4.36

In the year 2020, Dave was single and had a total income of \$70,000. He took a deduction of \$13,000 and had a tax credit of \$1800. Calculate the tax owed by Dave.

The answer is provided at the end of this section.

In Example 4.36, the effect of replacing a \$1000 deduction by a \$1000 credit was to reduce the tax owed by \$780. This is a significant reduction in taxes and highlights the benefits of tax credits.

The Dow

In the late nineteenth century, tips and gossip caused stock prices to move because solid information was hard to come by. This prompted Charles H. Dow to introduce the Dow Jones Industrial Average (DJIA) in May 1896 as a benchmark to gauge the state of the market. The original DJIA was simply the average price of 12 stocks

xPACIFICA/Corbis Documentary/Getty Images

The New York Stock Exchange.

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that Mr. Dow picked himself. Today the Dow, as it is often called, consists of 30 "blue-chip" U.S. stocks picked by the editors of the *Wall Street Journal*. For example, in June 2009 General Motors was removed from the list as it entered bankruptcy protection. Here is the list as of March 2021.

The 30 Dow Companies

- 3M Company
- American Express Company
- Amgen
- Apple Incorporated
- Boeing Company
- Caterpillar Incorporated
- Chevron Corporation
- Cisco Systems Incorporated
- Coca-Cola Company
- Dow, Inc.
- Goldman Sachs Group Incorporated
- Home Depot Incorporated
- Honeywell
- Intel Corporation
- International Business Machines Corporation
- Johnson & Johnson
- JPMorgan Chase & Company
- McDonald's Corporation
- Merck & Company Incorporated
- Microsoft Corporation
- Nike Incorporated
- Procter & Gamble Company
- Salesforce
- Travelers Companies Incorporated
- United Health Group Incorporated
- Verizon Communications Incorporated
- Visa Incorporated
- Walgreens Boots Alliance
- Wal-Mart Stores Incorporated
- Walt Disney Company

As we said earlier, the original DJIA was a true average—that is, you simply added up the stock prices of the 12 companies and divided by 12. In 1928 a divisor of 16.67 was used to adjust for mergers, takeovers, bankruptcies, stock splits, and company substitutions. Today, they add up the 30 stock prices and divide by 0.15198707565833, or equivalently, multiply by 1/0.15198707565833 or about 6.58. This means that for every \$1 move in any Dow company's stock price, the average changes by about 6.58 points. (The DJIA is usually reported using two decimal places.)

EXAMPLE 4.37 Finding changes in the Dow: Disney stock goes up

Suppose the stock of Walt Disney increases in value by \$3 per share. If all other Dow stock prices remain unchanged, how does this affect the DJIA?

SOLUTION

Each \$1 increase causes the average to increase by about 6.58 points. So a \$3 increase would cause an increase of about $3 \times 6.58 = 19.74$ points in the Dow.

TRY IT YOURSELF 4.37

Suppose the stock of Microsoft decreases in value by \$4 per share. If all other Dow stock prices remain unchanged, how does this affect the DJIA?

The answer is provided at the end of this section.

The graph in **Figure 4.14** shows how the Dow has moved over the last several decades. In Exercises 30 through 33, we explore a few of the more common types of stock transactions.

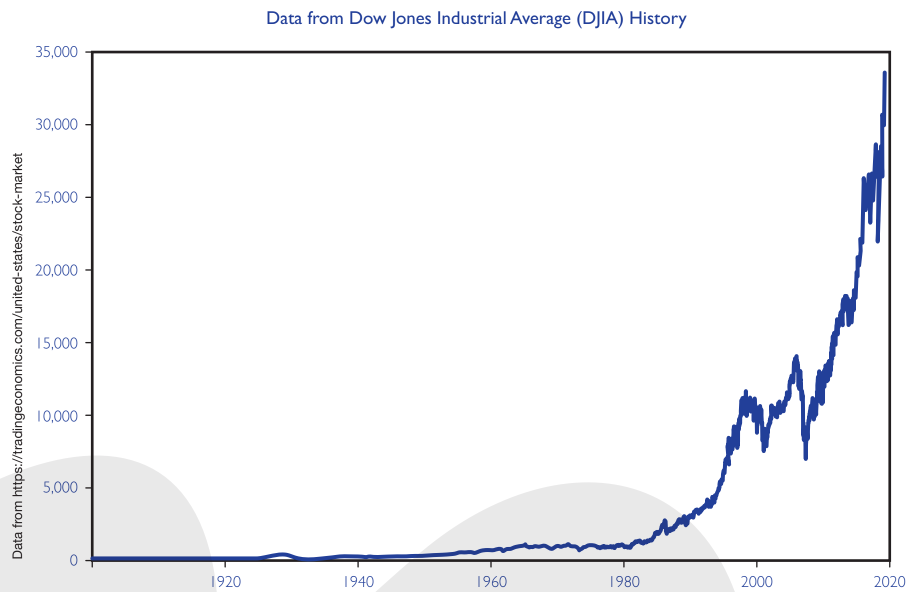

FIGURE 4.14 This graph shows the movement of the Dow.

ALGEBRAIC SPOTLIGHT 4.6 Derivation of the Buying Power Formula

Suppose the inflation rate is $i\%$ per year. We want to derive the buying power formula:

$$\text{Percent decrease in buying power} = \frac{100\, i}{100 + i}$$

Suppose we could buy a commodity, say, one pound of flour, for one dollar a year ago. An inflation rate of $i\%$ tells us that today that same pound of flour would cost $1 + i/100$ dollars. To find the new buying power of the dollar, we need to know how much flour we could buy today for one dollar. Because one pound of flour costs $1 + i/100$ dollars, one dollar will buy

$$\frac{1}{1 + i/100} = \frac{100}{100 + i} \text{ pounds of flour}$$

This quantity represents a decrease of

$$1 - \frac{100}{100 + i} = \frac{i}{100 + i} \text{ pounds}$$

from the one pound we could buy with one dollar a year ago. The percentage decrease in the amount of flour we can buy for one dollar is

$$\text{Percentage decrease} = \frac{\text{Decrease}}{\text{Previous value}} \times 100\% = \frac{\frac{i}{100+i}}{1} \times 100\% = \frac{100\, i}{100 + i}$$

This is the desired formula for the percentage decrease in buying power.

WHAT DO YOU THINK?

CPI and benefits: In the United States, some government entitlements, such as Social Security payments, are tied to the inflation rate. Explain why connecting retirement benefits to inflation makes sense. Find other financial arrangements that are tied to inflation.

Questions of this sort may invite many correct responses. We present one possibility: Inflation causes the cost of typical consumer items to rise. If income remains constant while inflation drives up prices, many who depend on Social Security will suffer. Increasing benefits to match inflation means that Social Security maintains a constant value to recipients. Other federal programs with benefits tied to inflation include military retirement, Veterans Disability Compensation, food stamps, Medicaid eligibility, and many more.

Further questions of this type may be found in the *What Do You Think?* section of exercises.

Try It Yourself answers

Try It Yourself 4.30: Calculating inflation: CPI increase to 225 4.7%.

Try It Yourself 4.31: Calculating decrease in buying power: 6500% inflation 98.5%.

Try It Yourself 4.32: Calculating inflation: 5.2% decrease in buying power 5.5%.

Try It Yourself 4.34: Calculating the tax: A single person $2202.50.

Try It Yourself 4.36: Comparing deductions and credits: Differing effects $6530.

Try It Yourself 4.37: Finding changes in the Dow: Microsoft goes down The DJIA decreases by 26.32 points.

Exercise Set 4.5

Test Your Understanding

1. The Consumer Price Index measures ______.
2. As inflation increases, the buying power of a dollar ______.
3. Explain in a sentence what the “marriage penalty” is.
4. The Dow Jones Industrial Average measures **a.** the value of selected stocks **b.** the growth of industry in America **c.** the gross national product.
5. True or false: To say you are in the 25% tax bracket means that your marginal tax rate is 25%.
6. The marginal tax rate measures **a.** the amount of taxes you owe **b.** the tax rate to which additional income is subject **c.** the percentage of your income that is paid in taxes **d.** none of the above.
7. The term “hyperinflation” refers to ______.
8. Explain how a tax deduction affects your income tax.

Problems

9. Calculating inflation. Suppose the CPI increases this year from 205 to 215. What is the rate of inflation for this year? Round your answer to the nearest tenth of a percent.

10. Calculating decrease in buying power. Suppose the rate of inflation this year is 3%. What is the percentage decrease in the buying power of a dollar? Round your answer to the nearest tenth of a percent.

11. Calculating inflation from decrease in buying power. Suppose the buying power of a dollar decreases by 4% this year. What is the rate of inflation this year? Round your answer to the nearest tenth of a percent.

12. Calculating tax. Use Table 4.6 to calculate the tax due from a married couple filing jointly with a taxable income of $107,000.

13. Deductions. If you earn $75,000 this year and have deductions of $17,000, what is your taxable income?

14. Changes in the DOW. Suppose the stock of the 3M Company increases by $5 per share while all other Dow stock prices remain the same. How does this affect the Dow Jones Industrial Average?

15. Large inflation rate. The largest annual rate of inflation in the CPI table, Table 4.3 on page 289, was 13.3% for the year 1979. What year saw the next largest rate? Why do you think the inflation rate in the table for 1949 is blank?

16. Food inflation. This exercise refers to *In the News 4.10*, which is from the Web site of the United States Department of Agriculture.

a. If your total food bill was $300 in October 2015, what was it in October 2016?

b. If your at-home food bill was $300 in October 2015, what was it in October 2016?

c. If your away-from-home food bill was $300 in October 2015, what was it in October 2016?

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IN THE NEWS 4.10

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Food Price Outlook, 2016

United States Department of Agriculture
Economic Research Service

The all-items Consumer Price Index (CPI), a measure of economy-wide inflation, rose 0.1 percent from September to October 2016 and is 1.6 percent above the October 2015 level. While the CPI for all food also rose 0.1 percent from September to October, food prices are 0.4 percent lower compared with the October 2015 level. The degree of food price inflation varies depending on whether the food was purchased for consumption away from home or at home.

- The food-away-from-home (restaurant purchases) CPI was up 0.1 percent in October and is 2.4 percent higher than October 2015; and
- The food-at-home (grocery store or supermarket food items) CPI was flat from September to October and is 2.3 percent lower than last October. Retail food prices have remained flat or decreased for 8 of the first ten months in 2016.

17. Inflation compounded. In this exercise, we see the cumulative effects of inflation. We refer to Table 4.3 on page 289.

a. Find the three-year inflation rate from December 1977 to December 1980.

b. Consider the one-year inflation rate for each of the three years from December 1977 to December 1980. Find the sum of these three numbers. Is the sum the same as your answer to part a?

18. More on compounding inflation. Here is a hypothetical CPI table.

Year	Hypothetical CPI	Inflation rate
1935	10	—
1936	20	%
1937	40	%
1938	80	%

a. Fill in the missing inflation rates.

b. Find the three-year inflation rate from 1935 to 1938.

c. Find the sum of the three inflation rates during the three years from 1935 to 1938. Is your answer here the same as your answer to part b?

d. Use the idea of compounding from Section 4.1 to explain the observation in part c.

19. Sudanese *pound*. Table 4.4 on page 290 shows that the inflation rate for Sudan in the year 2020 was 142%. By how much did the buying power of the Sudanese *pound* (the currency in Sudan) decrease during 2020?

20. Find the inflation rate. Suppose the buying power of a dollar went down by 60% over a period of time. What was the inflation rate during that period?

21. Continuing inflation. Suppose that prices increase 3% each year for 10 years. How much will a jacket that costs $80 today cost in 10 years? *Suggestion*: The price of the jacket increases by the same percentage each year, so the price is an exponential function of the time in years. You can think of the price as the balance in a savings account with an APY of 3% and an initial investment of $80.

22. More on continuing inflation. *This is a continuation of Exercise 21*. If prices increase 3% each year for 10 years, what is the percentage decrease in the buying power of currency over the 10-year period?

23. Flat tax. Steve Forbes ran for U.S. president in 1996 and 2000 on a platform proposing a 17% flat tax, that is, an income tax that would simply be 17% of each tax payer's taxable income. Suppose that Alice was single in the year 2020 with a taxable income of $30,000 and that Joe was single in the year 2020 with a taxable income of $300,000.

a. What was Alice's tax? Use the tax table on page 292.

b. What was Joe's tax? Use the tax table on page 292.

c. If the 17% flat tax proposed by Mr. Forbes had been in effect in 2020, what would Alice's tax have been?

d. What would Joe's tax have been under the 17% flat tax?

e. When you compare Alice and Joe, what do you think about the fairness of the flat tax versus a progressive tax?

24. More on the flat tax. Let's return to Alice and Joe from Exercise 23. We learn that Joe actually made $600,000, but his taxable income was only $300,000 because of various deductions allowed by the system in 2020. Proponents of the flat tax say that many of these deductions should be eliminated, so the 17% flat tax should be applied to Joe's entire $600,000.

a. What would Joe's tax be under the 17% flat tax?

b. How much more tax would Joe pay than under the 2020 system?

c. How much income would Joe have to make for the 17% flat tax to equal the amount he pays in the year 2020 with a taxable income of $300,000?

d. When you compare Alice and Joe now, what do you think about the fairness of the flat tax versus a progressive tax?

25. Bracket creep. At the start of 2020, your taxable income was $40,000, and you received a cost-of-living raise because of inflation. In 2020, inflation was 1.4% and your raise resulted in a 1.4% increase in your taxable income. By how much, and by what percent, did your taxes go up over what they would have been without a raise? (Assume that you were single in 2020.) *Remark*: Note that your buying power remains the same, but you're paying higher taxes. Not only that, but you're paying at a higher marginal rate! This phenomenon is known as "bracket creep," and federal tax tables are adjusted each year to account for this.

26. Deduction and credit. In the year 2020, Ethan was single and had a total income of $55,000. He took a deduction of $12,500 and had a tax credit of $1500. Calculate the tax owed by Ethan.

27. Moving DJIA. Suppose the stock of McDonald's increases in value by $2 per share. If all other Dow stock prices remain unchanged, how does this affect the DJIA?

28. Average price. What is the average price of a share of stock in the Dow list when the DJIA is 10,000? Remember that there are 30 companies.

29. Dow highlights. Use Figure 4.14 to determine in approximately what year the DJIA first reached 5000. About when did it first reach 10,000?

Exercises 30 through 33 are suitable for group work.

30–33. Stock market transactions. *There are any number of ways to make (or lose) money with stock market transactions. In Exercises 30 through 33, we will explore a few of the more common types of transactions. (Fees for such transactions will be ignored.)*

30. Market order. The simplest way to buy stock is the *market order*. Through your broker or online, you ask to buy 100 shares of stock X at market price. As soon as a seller is located, the transaction is completed at the prevailing price. That price will normally be very close to the latest quote, but the prevailing price may be different if the price fluctuates between the time you place the order and the time the transaction is completed. Suppose you place a market order for 100 shares of stock X and the transaction is completed at $44 per share. Two weeks later the stock value is $58 per share and you sell. What is your net profit?

31. Limit orders. If you want to insist on a fixed price for a transaction, you place a *limit order*. That is, you offer to buy (or sell) stock X at a certain price. If the stock can be purchased for that price, the transaction is completed. If not, no transaction occurs. Often limit orders have a certain expiration date. Suppose you place a limit order to buy 100 shares of stock X for $40 per share. When the stock purchase is completed, you immediately place a limit order to sell stock X at $52 per share. The following table shows the value of stock X. On which days are these two transactions completed, and what is your profit?

Date	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Market price	$44	$45	$40	$48	$52	$50

32. Stop loss and trailing stops. If you own a stock, a *stop-loss* order protects you from large losses. For example, if you own 100 shares of stock X, bought at $45 per share, you might place a stop-loss order for $40 per share. This order automatically sells your stock if the price drops to $40. No matter what happens, you can't lose more than $5 per share. A similar type of order that protects profits is the *trailing stop*. The trailing-stop order sells your stock if the value goes below a certain percentage of the market price. If the market price remains the same or drops, the trailing stop doesn't change and acts like a stop-loss order. If, however, the market price goes up, the trailing stop follows it so that it protects profits. Suppose, for example, that you own 100 shares of stock X, that the market price is $40 per share, and that you place a trailing-loss order of 5%. If the price drops by $2 (5% of $40), the stock is sold. If the market value increases to $44, then you will sell the stock when it declines by 5% of $44. That is, you sell when the market price drops to $41.80. Consider the table in Exercise 31, and suppose that you purchase 100 shares of the stock and place a 5% trailing-stop order on day 1. On which day (if any) will your stock be sold?

33. Selling short. *Selling short* is the selling of stock you do not actually own but promise to deliver. Suppose you place an order to sell short 100 shares of stock X at $35. Eventually, your order must be *covered*. That is, you must sell 100 shares of stock X at a value of $35 per share. If when the order is covered, the value of the stock is less than $35 per share, then you make money; otherwise, you lose money. Suppose that on the day you must cover the sell short order, the price of stock X is $50 per share. How much money did you lose?

The following exercise is designed to be solved using technology such as calculators or computer spreadsheets.

34. Mortgage interest deduction. Interest paid on a home mortgage is normally tax deductible. That is, you can subtract the total mortgage interest paid over the year in determining your taxable income. This is one advantage of buying a home. Suppose you take out a 30-year home mortgage for $250,000 at an APR of 8% compounded monthly.

a. Determine your monthly payment using the monthly payment formula in Section 4.2.

b. Make a spreadsheet showing, for each month of the first year of your payment, the amount that represents interest, the amount toward the principal, and the balance owed.

c. Use the results of part b to find the total interest paid over the first year. (Round your answer to the nearest dollar.) That is what you get to deduct from your taxable income.

d. Suppose that your marginal tax rate is 27%. What is your actual tax savings due to mortgage payments? Does this make the $250,000 home seem a bit less expensive?

Writing About Mathematics

35. Flat tax. Do research to determine the pros and cons of a flat tax.

36. History: More on selling short. In 1992 George Soros "broke the Bank of England" by selling short the British pound. Write a brief report on his profit and exactly how he managed to make it.

37. History: The Knights Templar. The Knights Templar was a monastic order of knights founded in 1112 to protect pilgrims traveling to the Holy Land. Recent popular novels have revived interest in them. Some have characterized the Knights Templar as the first true international bankers. Report on the international aspects of their early banking activities.

38. History: The stock market. The Dow Jones Industrial Average normally fluctuates, but over the last half-century it has generally increased. Dramatic drops in the Dow (stock market crashes) can have serious effects on the economy. Report on some of the most famous of these. Be sure to include the crash of 1929.

39. History: The Federal Reserve Bank. The Federal Reserve Bank is an independent agency that regulates various aspects of American currency. Write a report on the Federal Reserve Bank. Your report should include the circumstances of its creation.

40. History: The SEC. The Securities and Exchange Commission regulates stock market trading in the United States. Write a report on the creation and function of the SEC.

41. Low and high inflation. Find current inflation rates for some countries not mentioned in this book and write a report about your findings. In particular, look for countries with very low and very high inflation.

42. Other countries' stock markets. Other countries have their own stock markets. Examine some of them and write a report about your findings. Can you see significant differences in how they perform?

43. Stagflation. Look up the meaning of *stagflation*. There was a period in history when the United States suffered from it. Write a report about this period.

44. Laffer curve. Look up the meaning of the *Laffer curve* and write a report about it.

What Do You Think?

45. Hyperinflation. Explain what is meant by the term *hyperinflation*. Use the Internet to find recent examples. **Exercise 19 is relevant to this topic.**

46. Inflation and buying power. If the inflation rate doubles, does the percentage decrease in buying power also double? If not, is the new percentage decrease in buying power more than twice the old one or less than twice the old one? Interpret your answer.

47. The Dow. The Dow is based on the prices of 30 "blue-chip" U.S. stocks. Why is it not based on the prices of all U.S. stocks?

48. The logarithmic Dow. Read Exercises 43 and 44 in Section 3.3. Compare the graphs on logarithmic and linear scales for the stock price of Google. Do the same thing for Apple and another company you find interesting. Explain what you observe.

49. Good inflation. Inflation is usually considered to be a bad thing, but some economists have recently suggested that more of it might be good because it would help with the national debt. Can you explain how inflation could make it easier for the government to pay down the national debt?

50. Deflation. Deflation is sometimes called "negative inflation" because prices go down rather than up. Deflation is considered a potentially bad thing by many economists because consumers will put off buying things, hoping for lower prices, and this behavior causes the economy to slow down. Use the Internet to find some historical examples of deflation.

51. Marriage penalty. Explain what is meant by the *marriage penalty*. Is it a fair way to tax married couples? Explain your answer.

52. Tax deduction and tax credit. Explain the difference between a tax deduction and a tax credit. Use the Internet to help you find examples of each. **Exercise 26 is relevant to this topic.**

53. Stock market. For background information, read Exercises 30 through 33. Suppose that you expect the price of a certain stock to decline in the near future. If you decide to deal in that stock, which of the following types of transactions might be to your benefit: market order, limit order, stop loss, or selling short? Explain why you made the choice you did.

CHAPTER SUMMARY

This chapter is concerned with financial transactions of two basic kinds: saving and borrowing. We also consider important financial issues related to inflation, taxes, and the stock market.

Saving money: The power of compounding

The principal in a savings account typically grows by interest earned. Interest can be credited to a savings account in two ways: as *simple interest* or as *compound interest*. For simple interest, the formula for the interest earned is

$$\text{Simple interest earned} = \text{Principal} \times \text{Yearly interest rate (as a decimal)} \times \text{Time in years}$$

Financial institutions normally compound interest and advertise the *annual percentage rate* or APR. The interest rate for a given compounding period is calculated using

$$\text{Period interest rate} = \frac{\text{APR}}{\text{Number of periods in a year}}$$

We can calculate the account balance after t periods using the compound interest formula:

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1+r)^t$$

Here, r is the period interest rate expressed as a decimal, and it should not be rounded. In fact, it is best to do all the calculations and then round.

The *annual percentage yield* or APY is the actual percentage return in a year. It takes into account compounding of interest and is always at least as large as the APR. If n is the number of compounding periods per year,

$$\text{APY} = \left(1 + \frac{\text{APR}}{n}\right)^n - 1$$

Here, both the APR and the APY are in decimal form. The APY can be used to calculate the account balance after t years:

$$\text{Balance after } t \text{ years} = \text{Principal} \times (1 + \text{APY})^t$$

Here, the APY is in decimal form.

The *present value* of an investment is the amount we initially invest. The *future value* is the value of that investment at some specified time in the future. If the investment grows by compounding of interest, these two quantities are related by the compound interest formula. We can rearrange that formula to give the present value we need for a desired future value:

$$\text{Present value} = \frac{\text{Future value}}{(1+r)^t}$$

In this formula, t is the total number of compounding periods, and r is the period interest rate expressed as a decimal.

The *Rule of 72* can be used to estimate how long it will take for an account growing by compounding of interest to double in size. It says that the doubling time in years can be approximated by dividing 72 by the APR, where the APR is expressed as a percentage, not as a decimal. The exact doubling time can be found using the formula:

$$\text{Number of periods to double} = \frac{\log 2}{\log(1+r)}$$

Here, r is the period interest rate as a decimal.

Borrowing: How much car can you afford?

With an *installment loan*, you borrow money for a fixed period of time, called the *term* of the loan, and you make regular payments (usually monthly) to pay off the loan plus interest in that time. Loans for the purchase of a car or home are usually installment loans.

If you borrow an amount at a monthly interest rate r (as a decimal) with a term of t months, the monthly payment is

$$\text{Monthly payment} = \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)}$$

It is best to do all the calculations and then round. A companion formula tells how much you can borrow for a given monthly payment:

$$\text{Amount borrowed} = \frac{\text{Monthly payment} \times ((1+r)^t - 1)}{(r \times (1+r)^t)}$$

For all loans, the monthly payment is *at least* the amount we would pay each month if no interest were charged, which is the amount of the loan divided by the term (in months) of the loan. This number can be used to estimate the monthly payment for a short-term loan if the APR is not large. For all loans, the monthly payment is *at least* as large as the principal times the monthly interest rate as a decimal. This number can be used to estimate the monthly payment for a long-term loan with a moderate or high interest rate.

A record of the repayment of a loan is kept in an *amortization table*. In the case of buying a home, an important thing for the borrower to know is how much *equity* he or she has in the home. The equity is the total amount that has been paid toward the principal, and an amortization table keeps track of this amount.

Some home loans are in the form of an *adjustable-rate mortgage* or ARM, where the interest rate may vary over the life of the loan. For an ARM, the initial rate is often lower than the rate for a comparable fixed-rate mortgage, but rising rates may cause significant increases in the monthly payment.

Saving for the long term: Build that nest egg

Another way to save is to deposit a certain amount into your savings account at the end of each month. If the monthly interest rate is r as a decimal, your balance is given by

$$\text{Balance after } t \text{ deposits} = \frac{\text{Deposit} \times ((1+r)^t - 1)}{r}$$

The ending balance is often called the *future value* of this savings arrangement. A companion formula gives the amount we need to deposit regularly in order to achieve a goal:

$$\text{Needed deposit} = \frac{\text{Goal} \times r}{((1+r)^t - 1)}$$

Retirees typically draw money from their nest eggs in one of two ways: either as a *perpetuity* or as an *annuity*. An annuity reduces the principal over time, but a perpetuity does not. The principal (your nest egg) is often called the *present value*.

For an annuity with a term of t months, we have the formula

$$\text{Monthly annuity yield} = \frac{\text{Nest egg} \times r(1+r)^t}{((1+r)^t - 1)}$$

In this formula, r is the monthly interest rate as a decimal. A companion formula gives the nest egg needed to achieve a desired annuity yield:

$$\text{Nest egg needed} = \frac{\text{Annuity yield goal} \times ((1+r)^t - 1)}{(r(1+r)^t)}$$

Credit cards: Paying off consumer debt

Buying a car or a home usually involves a regular monthly payment that is computed as described earlier. But another way of borrowing is by credit card. If the balance is not paid off by the due date, the account is subject to finance charges. A simplified formula for the finance charge is

$$\text{Finance charge} = \frac{\text{APR}}{12} \times (\text{Previous balance} - \text{Payments} + \text{Purchases})$$

The new balance on the credit card statement is found as follows:

$$\text{New balance} = \text{Previous balance} - \text{Payments} + \text{Purchases} + \text{Finance charge}$$

Suppose we have a balance on our credit card and decide to stop charging. If we make only the minimum payment, the balance is given by the exponential formula

$$\text{Balance after } t \text{ minimum payments} = \text{Initial balance} \times ((1+r)(1-m))^t$$

In this formula, r is the monthly interest rate and m is the minimum monthly payment as a percent of the balance. Both r and m are in decimal form. The product $(1+r)(1-m)$ should not be rounded when the calculation is performed. Because the balance is a decreasing exponential function, the balance decreases very slowly in the long run.

Inflation, taxes, and stocks: Managing your money

The *Consumer Price Index* or CPI is a measure of the average price paid by urban consumers in the United States for a "market basket" of goods and services. The *rate of inflation* is measured by the percent change in the CPI over time.

Inflation reflects a decline of the buying power of the consumer's dollar. Here is a formula that tells how much the buying power of currency decreases for a given inflation rate i (expressed as a percent):

$$\text{Percent decrease in buying power} = \frac{100i}{100+i}$$

A key concept for understanding income taxes is the *marginal tax rate*. With a marginal tax rate of 30%, for example, the tax owed increases by \$0.30 for every \$1 increase in taxable income. Typically, those with a substantially higher taxable income have a higher marginal tax rate. To calculate our taxable income, we subtract any *deductions* from our total income. Then we can use the tax tables. To calculate the actual tax we owe, we subtract any *tax credits* from the tax determined by the tables.

The *Dow Jones Industrial Average* or DJIA is a measure of the value of leading stocks. It is found by adding the prices of 30 "blue-chip" stocks and dividing by a certain number, the *divisor*, to account for mergers, stock splits, and other factors. With the current divisor, for every \$1 move in any Dow company's stock price, the average changes by about 6.58 points.

KEY TERMS

CHAPTER QUIZ

1. We invest \$2400 in an account that pays simple interest of 8% each year. Find the interest earned after five years.

Answer \$960

If you had difficulty with this problem, see Example 4.1.

2. Suppose it were possible to invest \$8000 in a four-year CD that pays an APR of 5.5%.

a. What is the value of the mature CD if interest is compounded annually?

b. What is the value of the mature CD if interest is compounded monthly?

Answer **a.** \$9910.60, **b.** \$9963.60

If you had difficulty with this problem, see Example 4.3.

3. We have an account that pays an APR of 9.75%. If interest is compounded quarterly, find the APY. Round your answer as a percentage to two decimal places.

Answer 10.11%

If you had difficulty with this problem, see Example 4.4.

4. How much would you need to invest now in a savings account that pays an APR of 8% compounded monthly in order to have a future value of \$6000 in a year and a half?

Answer \$5323.64

If you had difficulty with this problem, see Example 4.7.

5. Suppose an account earns an APR of 5.5% compounded monthly. Estimate the doubling time using the Rule of 72, and then calculate the exact doubling time. Round your answers to one decimal place.

Answer Rule of 72: 13.1 years; exact method: 151.6 months (about 12 years and eight months)

If you had difficulty with this problem, see Example 4.8.

6. You need to borrow $6000 to buy a car. The dealer offers an APR of 9.25% to be paid off in monthly installments over $2\frac{1}{2}$ years.

a. What is your monthly payment?

b. How much total interest did you pay?

Answer **a.** $224.78, **b.** $743.40

If you had difficulty with this problem, see Example 4.11.

7. We can afford to make payments of $125 per month for two years for a used motorcycle. We're offered a loan at an APR of 11%. What price bike should we be shopping for?

Answer $2681.95

If you had difficulty with this problem, see Example 4.10.

8. Suppose we have a savings account earning 6.25% APR. We deposit $15 into the account at the end of each month. What is the account balance after eight years?

Answer $1862.16

If you had difficulty with this problem, see Example 4.21.

9. Suppose we have a savings account earning 5.5% APR. We need to have $2000 at the end of seven years. How much should we deposit each month to attain this goal?

Answer $19.57

If you had difficulty with this problem, see Example 4.22.

10. Suppose we have a nest egg of $400,000 with an APR of 5% compounded monthly. Find the monthly yield for a 10-year annuity.

Answer $4242.62

If you had difficulty with this problem, see Example 4.24.

11. Suppose your MasterCard calculates finance charges using an APR of 16.5%. Your previous statement showed a balance of $400, toward which you made a payment of $100. You then bought $200 worth of clothes, which you charged to your card. Complete the following table:

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1					

Answer

	Previous balance	Payments	Purchases	Finance charge	New balance
Month 1	$400.00	$100.00	$200.00	$6.88	$506.88

If you had difficulty with this problem, see Example 4.26.

12. Suppose your MasterCard calculates finance charges using an APR of 16.5%. Your statement shows a balance of $900, and your minimum monthly payment is 6% of that month's balance.

a. What is your balance after a year and a half if you make no more charges and make only the minimum payment?

b. How long will it take to get your balance under $100?

Answer **a.** $377.83, **b.** 46 monthly payments

If you had difficulty with this problem, see Example 4.29.

13. Suppose the CPI increases this year from 210 to 218. What is the rate of inflation for this year?

Answer 3.8%

If you had difficulty with this problem, see Example 4.30.

14. Suppose the rate of inflation last year was 20%. What was the percentage decrease in the buying power of currency over that year?

Answer 16.7%

If you had difficulty with this problem, see Example 4.31.